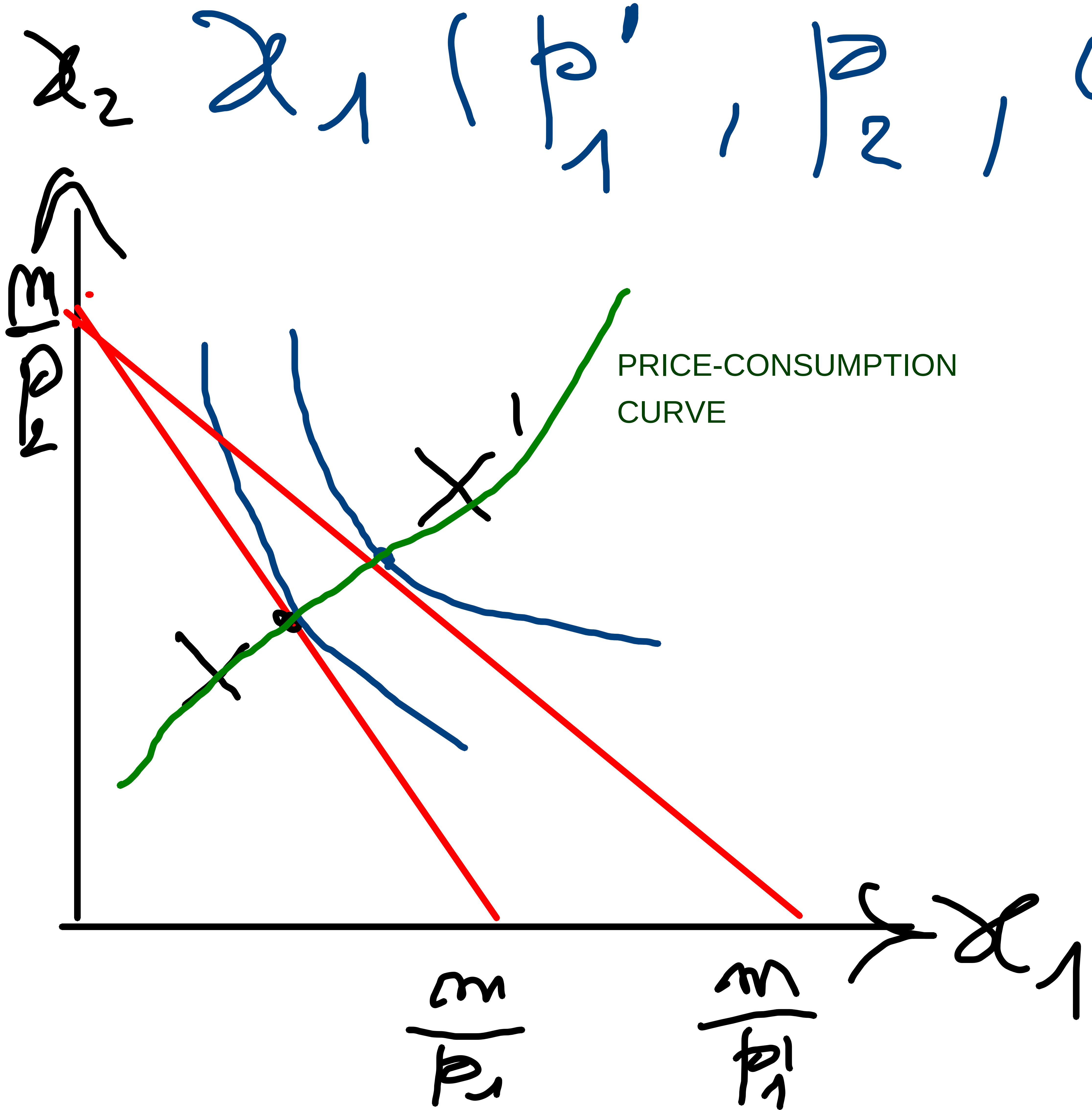
DEMAND OF GOOD 1 WHEN THE PRICE OF GOOD 1 IS CHANGING, m and p2 constant

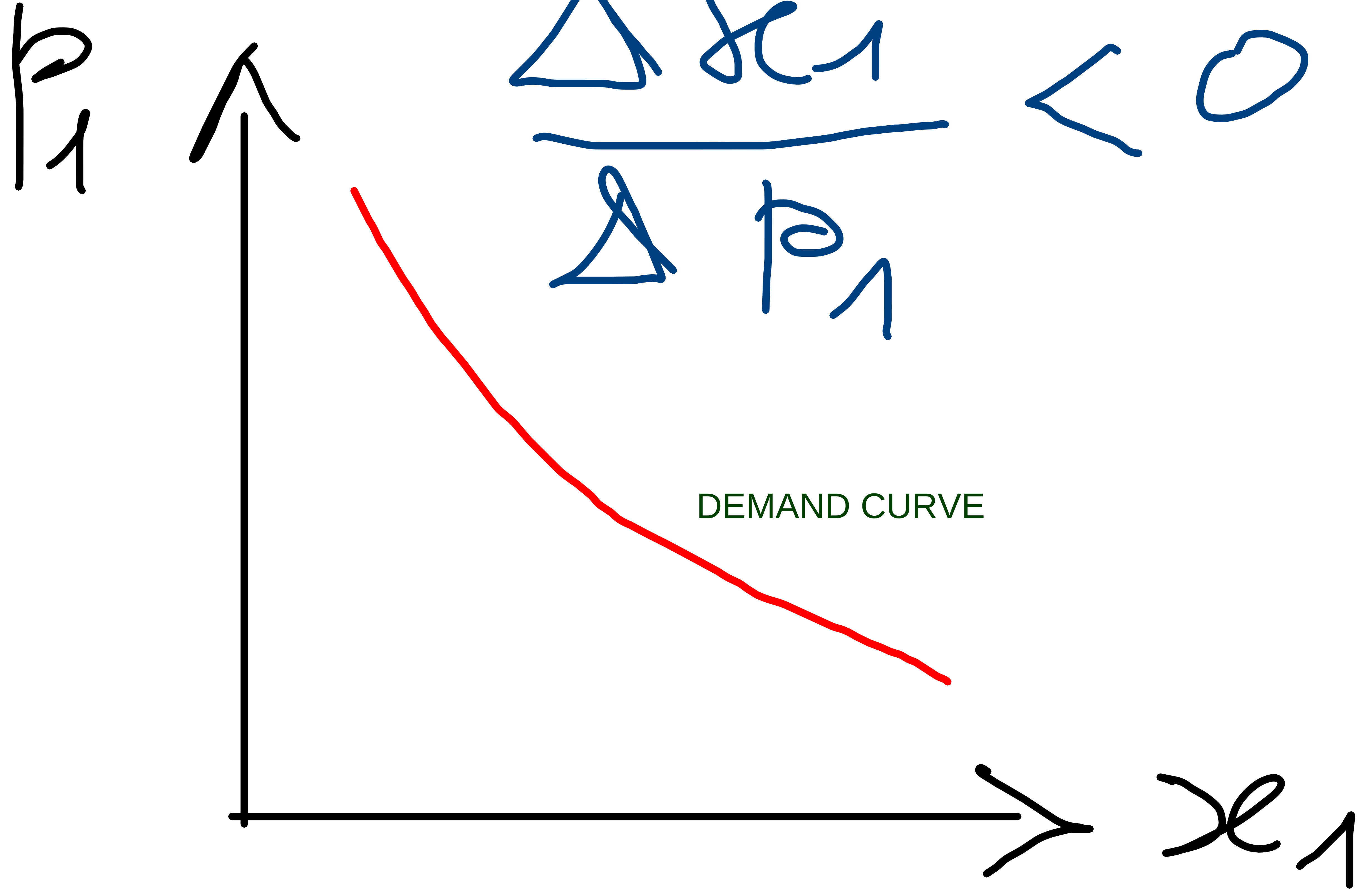
$x_1(p_1, p_2, m)$ $\qquad$ $p_1' < p_1$

$x_1(p_1', p_2, m)$ $\qquad$ $p_1' - p_1 = \Delta p_1 < 0$

$x_1$ is ORDINARY GOOD

$$\frac{\Delta x_1}{\Delta p_1} < 0$$



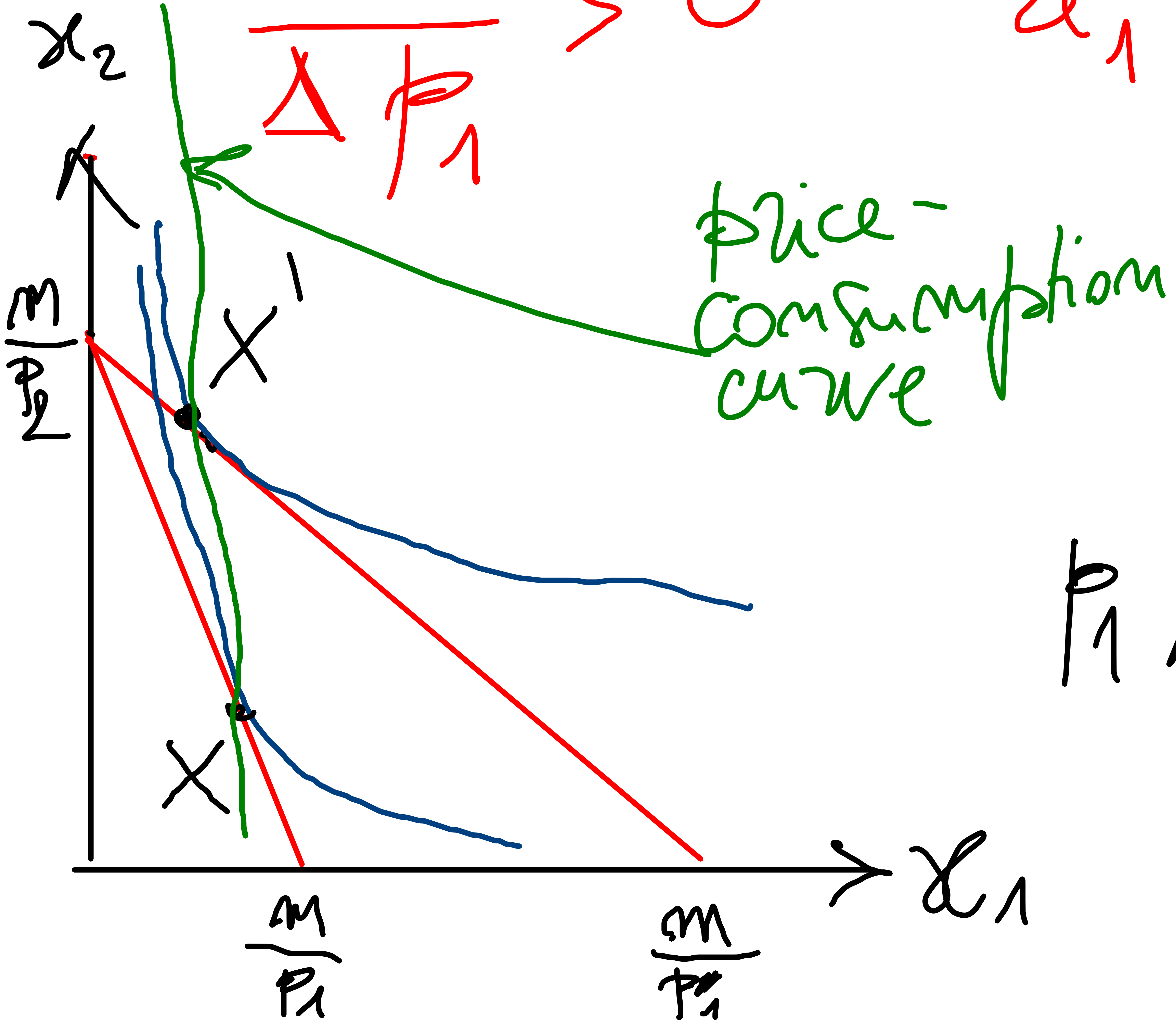
$$\frac{\Delta x_1}{\Delta p_1} > 0$$

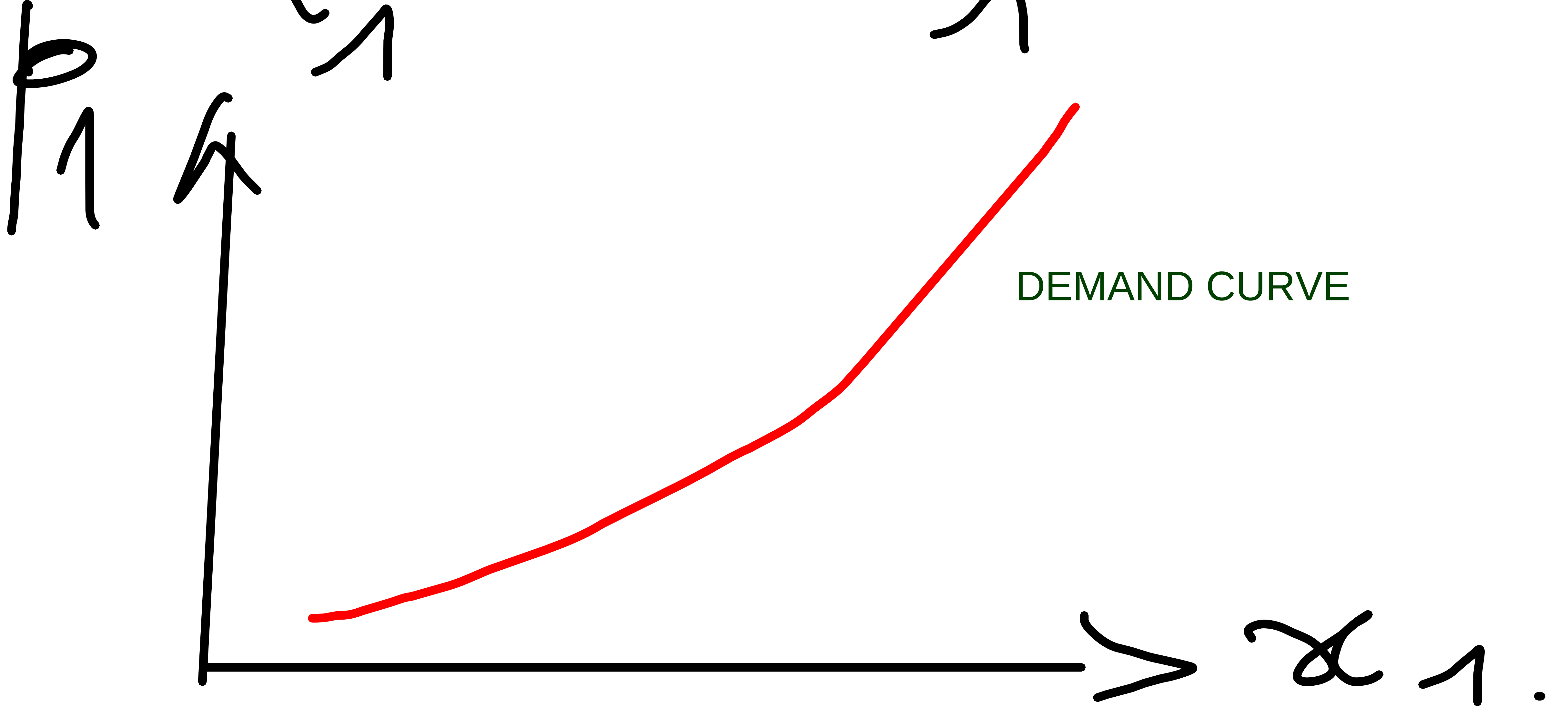
$x_1$ is GIFFEN GOOD

$m$ $p_2$ constant

$p_1' < p_1$

$x_1' < x_1$



ASSUME $x_1$ IS ORDINARY

$$x_2(p_1, p_2, m) \qquad x_2(p_1', p_2, m)$$

$$\Delta p_1 = p_1' - p_1$$

WHAT CAN WE SAY ON THE SIGN OF THE RATIO:

$$\frac{\Delta x_2}{\Delta p_1} ? \qquad \frac{\Delta x_1}{\Delta p_1} < 0$$

THIS FOLLOWS FROM THE FACT THAT x1 IS ORDINARY GOOD

if $x_1$ and $x_2$ are complements $\frac{\Delta x_2}{\Delta p_1} < 0$

if $x_1$ and $x_2$ are substitutes $\frac{\Delta x_2}{\Delta p_1} > 0$

there

if there are more then 2 goods

WE MUST CONSIDER ALL POSSIBLE INTERCATIONS AMONG THE DIFFERENT GOODS, NOT JUST THE INTERACIONS BETWEEN x1 AND x2

IF $\frac{\Delta x_2}{\Delta p_1} < 0$ $x_1, x_2$ ARE gross complements

IF $\frac{\Delta x_2}{\Delta p_1} > 0$ $x_1, x_2$ are gross substitutes

SUPPOSE WE OBSERVE THE CHOICES MADE BY A CONSUMER IN DIFFERENT PRICE AND INCOME SITUATIONS.

$$(x_1^*, x_2^*) \qquad (p_1, p_2)$$

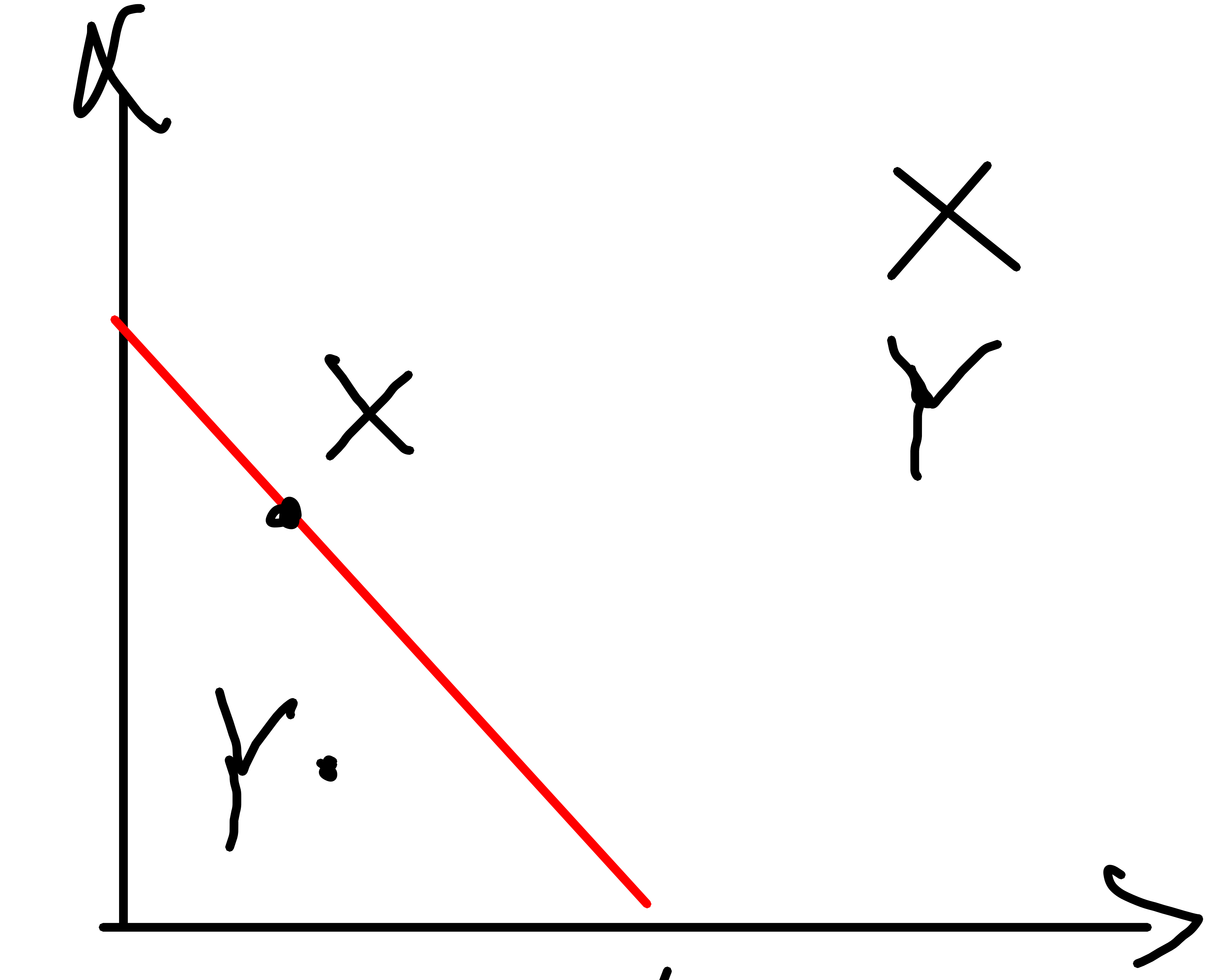
$$(y_1^*, y_2^*) \qquad (p_1', p_2')$$

$$\vdots \qquad \vdots$$

$$(z_1^*, z_2^*) \qquad (p_1^m, p_2^m)$$

WE ASK: are these choices consistent with utility maximization?

# PRINCIPLE OF REVEALED PREFERENCE




X chosen at prices $p_1, p_2$

Y chosen at prices $p_1', p_2'$

$$X \neq Y$$

$$x_1 p_1 + x_2 p_2 = m$$

$$y_1 p_1 + y_2 p_2 \leq m$$

X is directly revealed preferred to Y

IF Y WAS AFFORDABLE WHEN THE CHOICE WAS X

# Principle of Revealed Preference

if $(x_1, x_2)$ is the choice at $(p_1, p_2)$
and $(y_1, y_2) \neq (x_1, x_2)$ meets:

$$x_1 p_1 + x_2 p_2 \geq y_1 p_1 + y_2 p_2$$ then

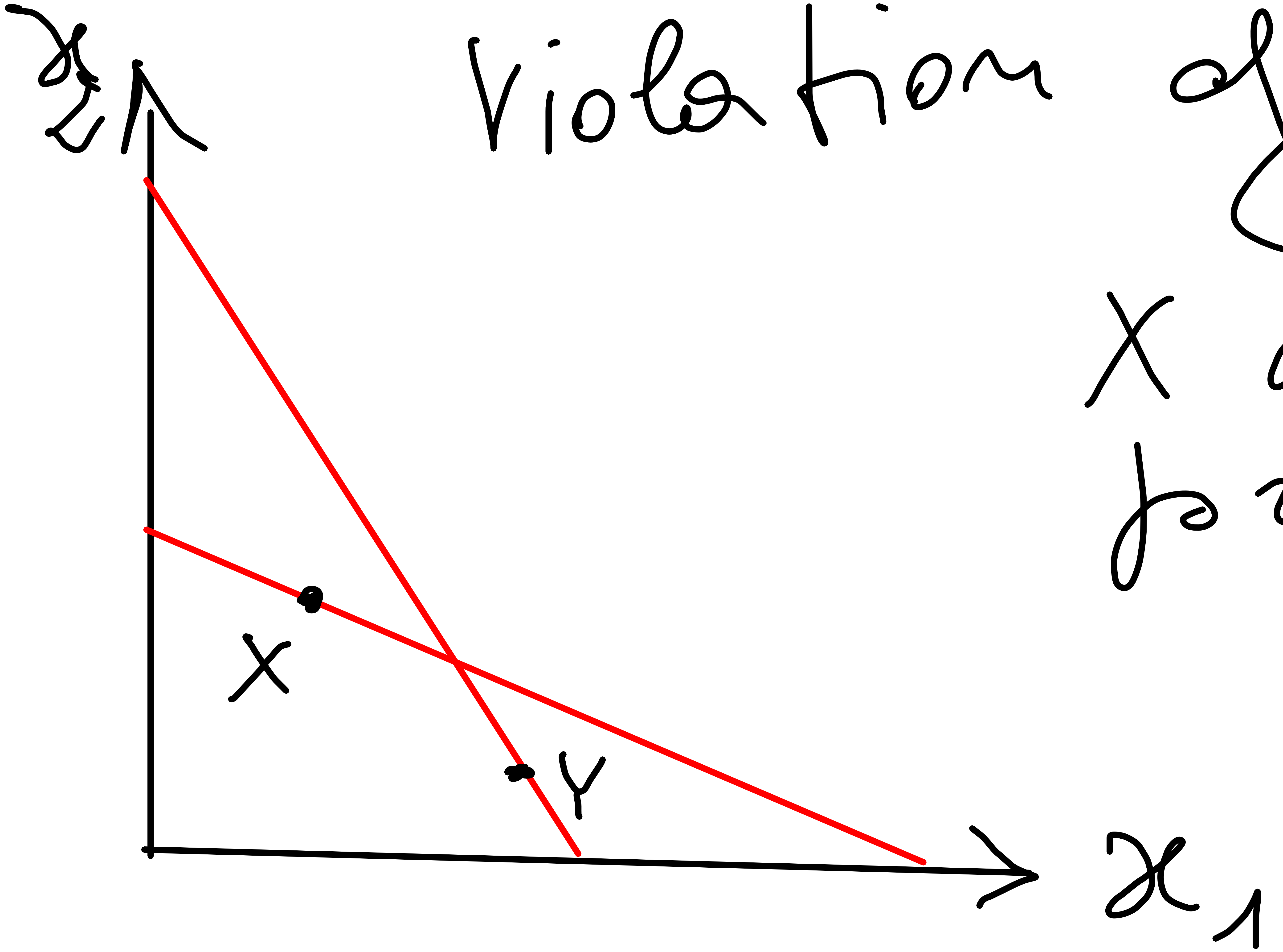
$$(x_1, x_2) \succ (y_1, y_2)$$

$(x_1, x_2)$ is revealed directly preferred to $(y_1, y_2)$

WARP: Weak axiom of revealed Pref.

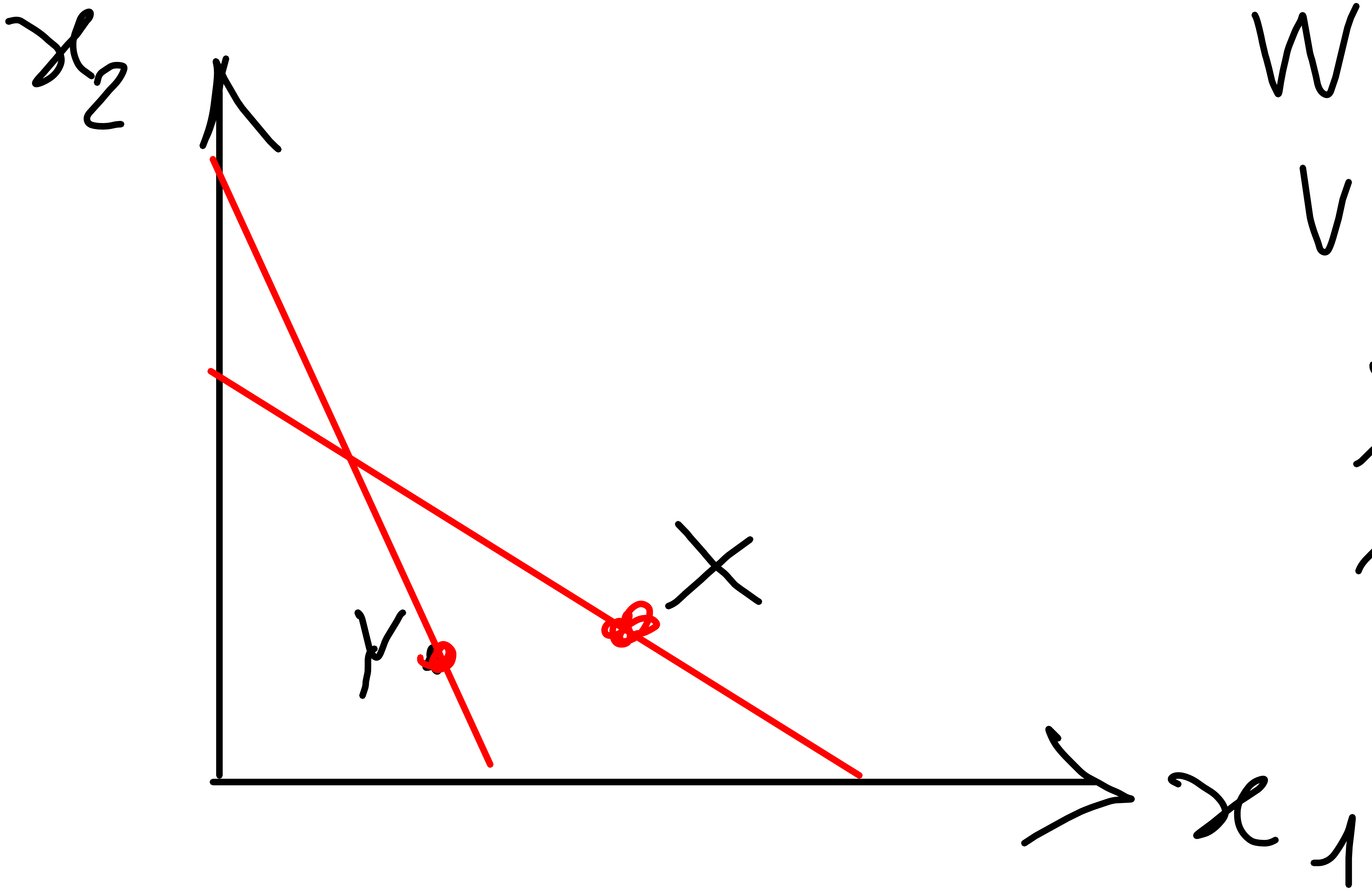
if $X = (x_1, x_2)$ is directly revealed preferred to $Y = (y_1, y_2) \neq X$, then it cannot be the case that $Y$ is directly revealed preferred to $X$.

Violation of WARP




X directly revealed preferred to Y

Y is directly revealed preferred to X



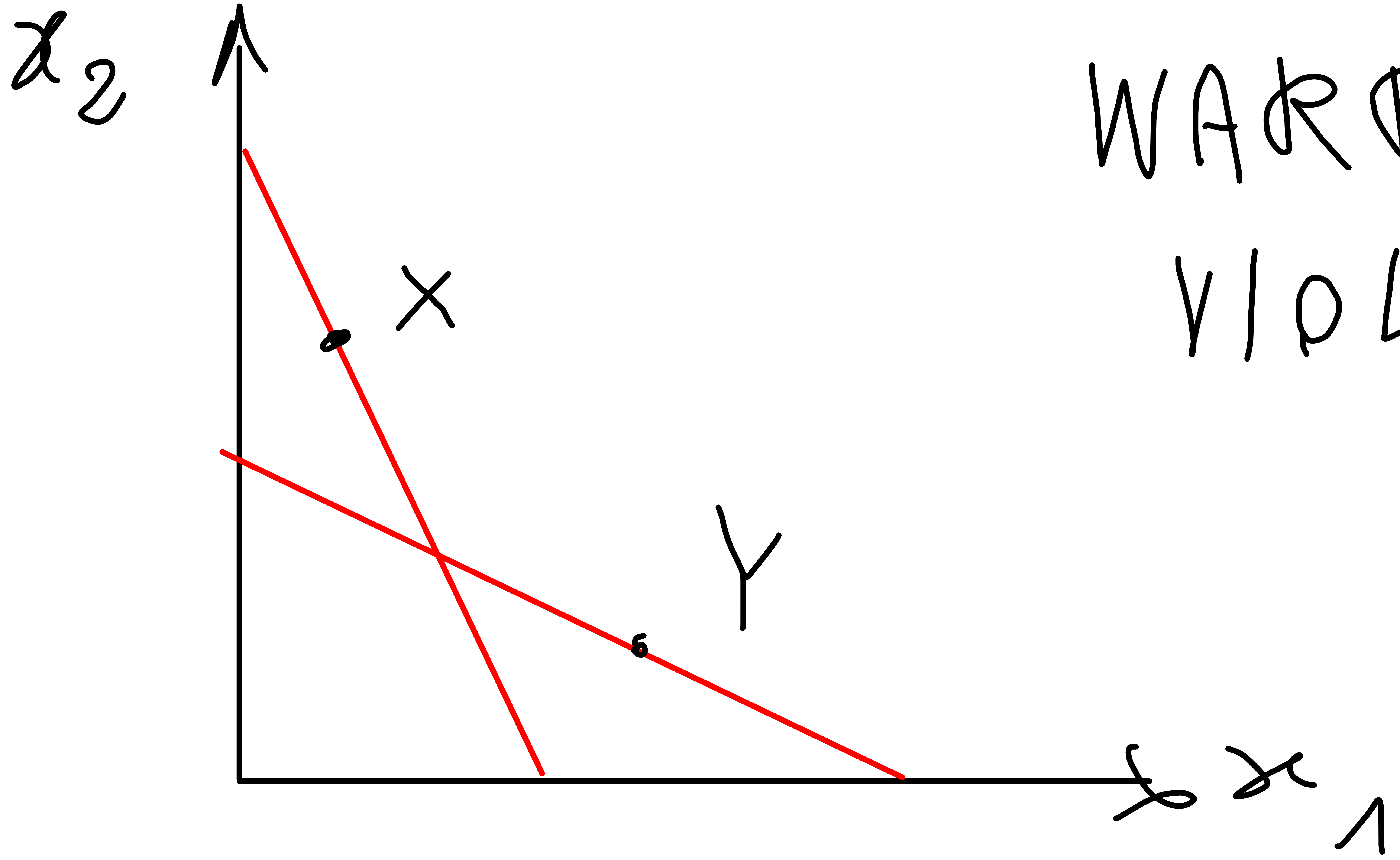

WARP NOT VIOLATED

$X \succ Y$

X directly revealed preferred to Y

$$y_1 p_1^1 + y_2 p_2^1 < x_1 p_1^1 + x_2 p_2^1$$

Y is not revealed preferred to X




WARP NOT VIOLATED

$$1\ \begin{cases} X = (3, 2) \\ p = (1, 1) \end{cases} \qquad 2\ \begin{cases} Y = (2, 3) \\ p' = (2, 1) \end{cases}$$

WAPP IS NOT VIOLATED

$$1\ \begin{cases} m^1 = p_1 x_1 + p_2 x_2 = 3 \cdot 1 + 2 \cdot 1 = 5 \\ \quad p_1 y_1 + p_2 y_2 = 2 \cdot 1 + 3 \cdot 1 = 5 \\ X \text{ directly revealed preferred to } Y \end{cases}$$

$$2\ \begin{cases} m^2 = p'_1 \cdot y_1 + p'_2 \cdot y_2 = 2 \cdot 2 + 3 \cdot 1 = 7 \qquad p'_1 x_1 + p'_2 x_2 = 8 > 7 \\ Y \text{ IS not DIRECTLY revealed to preferred to } X \end{cases}$$

NOW SUPPOSE WE HAVE THE FOLLOWING DATA DISCRIBING 3 CHOICE SITUATIONS: BUNDELES X1, X2, X3 WERE CHOSEN AT PRICE VECTORS p1, p2, p3 RESPECTIVELY

Set of choices:

PRICE VECTORS:

(1) $x^1 = (1, 0, 0)$ $\quad p^1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\quad 1 = x^1 p^1 = m^1$

income if choice is X1

$$x^1 p^1 = 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 2 = 1$$

(2) $x^2 = (0, 1, 0)$ $\quad p^2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $\quad x^2 p^2 = m^2 = 1$

income if choice is X2

(3) $x^3 = (0, 0, 1)$ $\quad p^3 = \begin{pmatrix} 1 \\ 2 \\ 1.5 \end{pmatrix}$ $\quad x^3 p^3 = m^3 = 1.5$

income if choice is X3

we now cpmpare situations 1 and 2

| | $X^1$ | $X^2$ |
|---|---|---|
| $p^1$ | 1 money income in 1 | 1 cost of X2 at prices p1 |
| $p^2$ | 2 cost of X1 at prices p2 | 1 money income in 2 |

$X^1 = (1, 0, 0)$

$X^2 = (0, 1, 0)$

$p^1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$p^2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

X1 is revealed directly preferred to X2

$$X^2 \cdot p^1 = 0 \cdot 1 + 1 \cdot 1 + 0 \cdot 2 = 1$$

$$X^1 \cdot p^2 = 1 \cdot 2 + 0 \cdot 1 + 0 \cdot 1 = 2$$

$X^1 \succ X^2$

WARP not violated

X2 is NOT revealed directly preferred to X1, therefore WARP is not violated

NOW COMPARE SITUATIONS 2 AND 3:

$$x^2 = (0, 1, 0) \qquad x^3 = (0, 0, 1)$$

$$p^2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \qquad p^3 = \begin{pmatrix} 1 \\ 2 \\ 1.5 \end{pmatrix}$$

| | $x^2$ | $x^3$ |
|---|---|---|
| $p^2$ | 1<br>MONEY INCOME IN 2 | 1<br>cost of X3 AT PRICES p2 |
| $p^3$ | 2<br>COST OF X2 AT PRICES p3 | 1.5<br>MONEY INCOME IN 3 |

$$x^3 \cdot p^2 = 0 \cdot 2 + 0 \cdot 1 + 1 \cdot 1 = 1$$

$$x^2 \succ x^3 \qquad x^2 \cdot p^3 = 0 \cdot 1 + 1 \cdot 2 + 0 \cdot 1.5 = 2$$

x2 revealed directly preferred to X3

$x^3$ not revealed preferred to $x^2$

WARP NOT VIOLATED

$$x^1 = (1, 0, 0) \qquad x^3 = (0, 0, 1)$$

$$p^1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad p^3 = \begin{pmatrix} 1 \\ 2 \\ 1.5 \end{pmatrix}$$

| | $x^1$ | $x^3$ |
|---|---|---|
| $p^1$ | 1 | 2 |
| $p^3$ | 1 | 1.5 |

$$x^3 p^1 = 0 \cdot 1 + 0 \cdot 1 + 1 \cdot 2 = 2$$

$$x^1 \cdot p^3 = 1 \cdot 1 = 1$$

$x^1$ is not <u>directly</u> revealed preferred to $x^3$

$$x^3 > x^1$$

WARP NOT VIOLATED

X3 IS REVEALED DIRECTLY PREFERRED TO X1

IF WE DROW TOGETHER OUR CONCLUSIONS WE DISCOVER THAT OUR CONSUMER DOES NOT HAVE TRANSITIVE PREFERENCES!

$x^1 > x^2$ WARP not violated

$x^2 > x^3$ " " "

$x^3 > x^1$ " " "

BUT OUR THEORY PRESCIBES THAT PREFERENCES ARE TRANSITIVE
IN THIS CASE:

$$x^1 > x^2 > x^3$$

contradiction!

$x^1$ indirectly revealed preferred to $x^3$

OR EQUIVALENTLY: X1 IS REVEALED INDIRECTLY PREFERRED TO X3

THE CONCLUSION IS THAT, TAKEN TOGETHER, CONSUMER CHOICES X1, X2, X3 CANNOT BE EXPLAINED (RATIONALIZED) BY A THOERY ASSUMING THAT PREFERENCES ARE TRANSITIVE, NOTWITHSTANDING THE FACT THAT WARP IS NOT VIOLATED

THE NECESSARY AND SUFFICIENT CONDITION IN ORDER THAT A SET OF CHOICES IS EXPLAINED BY MAXIMIZATION OF WELL-BEHAVED UTILITY FUNCTION IS:

STRONG AXIOM OF
REVEALED PREFERENCE

if X is directly or indirectly
revealed preferred to $Y \neq X$
then Y cannot be directly or
indirectly revealed preferred to X

EQUIVALENTLY: IF X IS REVEALED DIRECTLY OR INDIRECTLY PREFERRED TO Y, AND Y IS DIFFERENT FROM X, THAN IT CANNOT BE THE CASE THAT Y IS REVEALED DIRECTLY OR INDIRECTLY PREFERRED TO X.

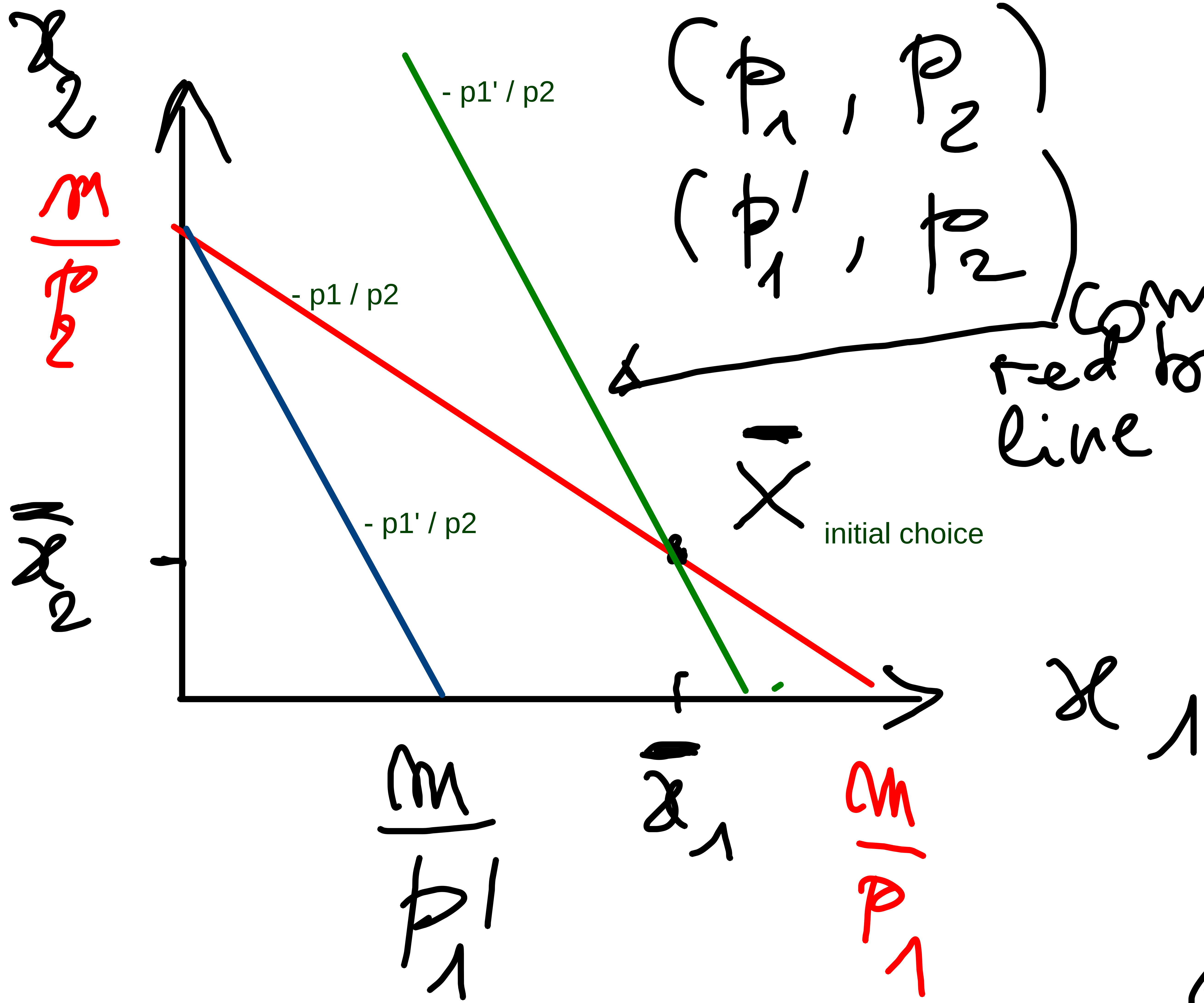
# SLUTSKY

$p_2$ $m$ given and constant

$\Delta p_1 = p_1' - p_1$

$\Delta p_1 \rightarrow \Delta\left(\frac{p_1}{p_2}\right)$ relative price change

$\Delta p_1 \rightarrow \Delta\left(\frac{m}{p_1}\right)$ change in purchasing power




$$p_1' - p_1 > 0$$

$$m' = \bar{x}_1 p_1' + \bar{x}_2 p_2$$

compensated income

$$m = \bar{x}_1 p_1 + \bar{x}_2 p_2$$

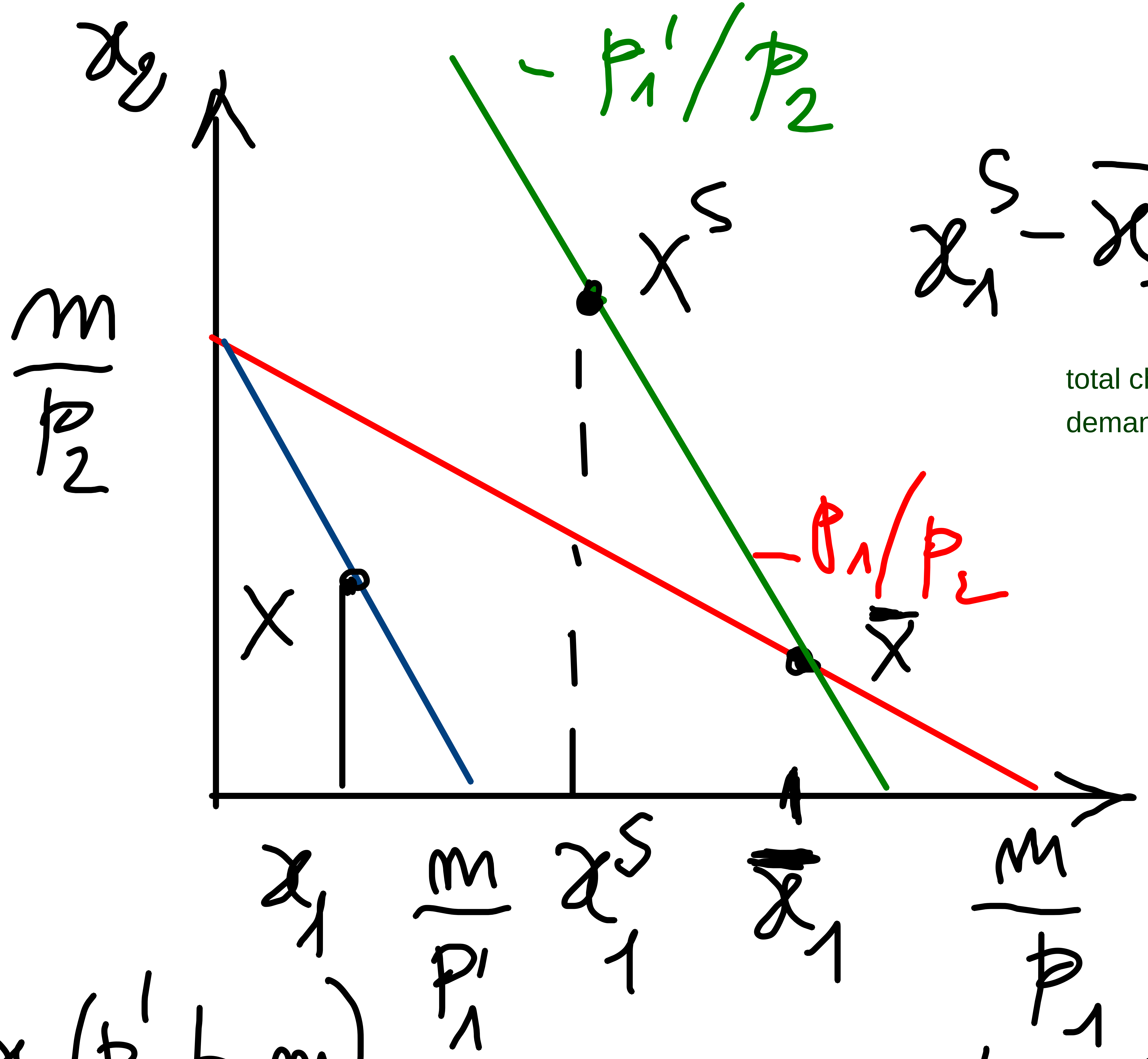
income

$$m' - m = \bar{x}_1 (p_1' - p_1)$$

income compensation

$m' =$ compensated income

$m' - m = \Delta m =$ income compensation $= \bar{x}_1 \cdot \Delta p_1$




$m'$ $p_1'$ $p_2$

$x_1^s - \bar{x}_1 =$ substitution effect

total change in demand $\Delta x_1 = x_1 - x_1$

$x_1^s - \bar{x}_1$ = CHANGE IN DEMAND caused by change in p1/p2 at constant purchasing power

$\bar{x}_1 = x_1(p_1, p_2, m)$

$x_1 = x_1(p_1', p_2, m)$

$x_1^s = x_1(p_1', p_2, m')$

compensated demand $x_1^s = x_1(p_1', p_2, m')$

$x_1 - x_1^s =$ income effect = change in demand caused by change in purchasing power