Asymmetric information in finance

Financial contract

Is a contract agreed upon by a borrower and a lender.
The contract is signed ***only if*** both parties' participation constraint is fulfilled:

Participation constraint:
expected return from the contract $\geq$ expected return from best alternative course of action (**opportunity cost**)

Example

An entrepreneur (borrower) is considering the following investment project:

Risky Investment at time t: $I = 100$

Cash Flow at time $t+1$: if success $CF_s = 300$ if failure: $CF_f = 0$

Probabilities: $\alpha = 0.7$ $1 - \alpha = 0.3$

Expected value (cash flow) $EV(I) = \alpha\, CF_s + (1 - \alpha)\, CF_f = 210$

Loan contract $L = 100$

Lender's opportunity cost: forgone opportunity to invest L = 100 in bonds at risk-free interest rate r

risk free interest rate $\boldsymbol{r} = 10\%$.

Lender's opportunity cost = $(1 + r)L = 110$

Limited liability: contractual clause that, in case the investment fails, the borrower is not forced to pay the capital and interest owed by using personal assets. Under this clause, the financial contract is risky for the bank.

Lender's participation constraint: the interest rate r_L on the loan L fixed by the bank must be sufficient to yield:

expected revenue from loan $L \geq$ opportunity cost $(1 + r)L$

Bank's expected revenue from L:

$$\alpha(1 + r_L)L + (1 - \alpha) min\ [(1 + r_L)L, CF_f] = \alpha(1 + r_L)L$$

Lender's participation constraint: $\alpha(1 + r_L)L \geq (1 + r)L$

Minimum risk adjusted interest factor: $1 + r_L \geq \frac{1+r}{\alpha} = \frac{1+0.1}{0.7} = 1.57$

$$r_L \geq 0.57$$

Remark: whenever $min\ [(1 + r_L)L, CF_f] < (1 + r)L$, *we have* $r_L > r$

Assume $r_L = \min r_L = 0.57$

By fixing the minimum risk-adjusted interest rate, the lender is indifferent between the risky contract and the sure asset. Competition on the credit market drives banks to fix the lowest risk-adjusted interest rate.

Borrower expected profit:

$E\pi = \alpha[CF_s - (1 + r_L)L] = EV - \alpha(1 + r_L)L$ = 210 – 0.7[1.57(100)] = 100.1 $\approx$ 100

Assumption: Borrower and lender are risk neutral

The financial contract is signed because participation constraint is fulfilled for both parties.

Borrower: $E\pi$ **> 0 and she has no other investment opportunity**

Lender: $\alpha(1 + r_L)L = (1 + r)L$

Forms of asymmetric information in financial contracts:

Hidden information: (pre-contractual opportunism)
The borrower overstates the true success probability α of the project, in the attempt of obtaining a lower risk adjusted interest rate r_L.

Hidden action: (post-contractual opportunism)

- the borrower mis-reports the true ex-post cash flow, in the attempt of avoiding the payment of $(1 + r_L)L$
- the borrower invests the money L received by the lender on a riskier investment project than was agreed with the bank (moral hazard).

Hidden information: adverse selection

Two types A, B of productive investment projects of size (cost) $L_A = L_B = L$

Frequency p_A, p_B with which projects of type A and B are selected is common information

$$EV_A = \alpha_A\, CF_{s,A} + (1 - \alpha_A)\, CF_{f,A} = \alpha_A\, CF_{s,A} \qquad CF_{f,A} = 0$$
$$EV_B = \alpha_B\, CF_{s,B} + (1 - \alpha_B)\, CF_{f,B} = \alpha_B\, CF_{s,B} \qquad CF_{f,B} = 0$$

Assume: $EV_A = EV_B$ and $CF_{s,B} > CF_{s,A}$ and $\alpha_A > \alpha_B$

⟶ **Project B is riskier than Project A**

The minimum risk adjusted interest rate is higher for the risky project:

$$\mathbf{1 + r_{L,A} = \frac{1+r}{\alpha_A} < \frac{1+r}{\alpha_B} = 1 + r_{L,B}}$$

The expected payment to the lender is the same $(1 + r)L$, under both types.

Hidden information:
Type B entrepreneurs hide their type to the bank, in the attempt of getting a lower interest charge. The bank is indeed unable to distinguish type A and B.

The bank charges the same risk adjusted interest factor (1 + r_L) to every borrower.

(1 + r_L) is so fixed that:

$(1 + r) = (p_A\, \alpha_A + p_B\, \alpha_B)(\mathbf{1} + \boldsymbol{r_L}) = \boldsymbol{p_s}(\mathbf{1} + \boldsymbol{r_L})$

$\boldsymbol{p_s} = (p_A\, \alpha_A + p_B\, \alpha_B) =$ ***average probability of success***

$(1 + r) = (\mathbf{1} + \boldsymbol{r_L})(p_A\, \alpha_A + p_B\, \alpha_B)$

$$\mathbf{1} + \boldsymbol{r_L} = \frac{\mathbf{1} + \boldsymbol{r}}{\boldsymbol{p_A \alpha_A + p_B \alpha_B}} = \frac{\mathbf{1} + \boldsymbol{r}}{\boldsymbol{p_s}}$$

By definition: $\boldsymbol{\alpha_A > p_S > \alpha_B}$ ***which implies:*** $\boldsymbol{r_{L,A} < r_L < r_{L,B}}$

hidden information implies:

low-risk (A), high-risk (B) projects are charged the same interest rate r_L.

in this example A and B yield the same expected cash flow $EV_A = EV_B$

This implies that if A and B pay the same interest rate r_L, the risky project B yields a higher expected profit than the safer project A.

This is because the probability of failure is higher for B, hence the expected repayment to the bank is lower, since in case of failure the payment is zero.

$$E\pi_A = \alpha_A\left[CF_{s,A} - (1 + r_L)L\right] = EV_A - \alpha_A(1 + r_L)L$$

$$E\pi_B = \alpha_B\left[CF_{s,B} - (1 + r_L)L\right] = EV_B - \alpha_B(1 + r_L)L$$

$EV_A = EV_B$ *and* $\alpha_A > \alpha_B \longrightarrow E\pi_A < E\pi_B$ *at any* r_L

Adverse selection: At interest rate $\boldsymbol{r_L} > \boldsymbol{r_{L,A}}$ entrepreneurs have an incentive to select type B projects.

The outcome is that type A projects disappear from the market, that is there is an endogenous change in the frequencies of types A and B:

$$\boldsymbol{p_A = 0}, \rightarrow \boldsymbol{p_B = 1} \text{ and } \boldsymbol{1 + r_L = 1 + r_{L,B}}$$

Remark:
Te previous example shows that:

- if the safe and the risky project yield the same expected cash flow,
- if the project type is hidden information, so that a uniform ‘average’ risk adjusted interest rate is fixed by the lender (to meet its participation constraint),
- If the limited liability clause applies,

  then:
  the risky project yields a higher expected return than the safe one, to the effect that adverse selection will take place.

Post contractual opportunism in finance

We now remove the assumption that project's type is hidden information. Investors fully disclose the project characteristics to the lender.

We also remove the assumption that the risky and safe projects yield the same expected cash flow. Instead, we assume that the safe project (that is the project with higher success probability) yields a higher expected cash flow, though its cash flow in case of success is lower.

We want to discuss the contractual implications of lender's imperfect monitoring of the actions carried out by the borrower after the contract is signed.

Hidden action: moral hazard

r_L = risk adjusted interest rate fixed by the bank

l = risky project α_l = success probability of l

h = safe project α_h = success probability of h

L = *cost of project l and h*

$CF_{h,s}$ = *cash flow if success* $CF_{h,f} = 0$ = *cash flow if failure*

$CF_{l,s}$ = *cash flow if success* $CF_{l,f} = 0$ = *cash flow if failure*

Assume safe project yields lower cash flow, if success, but succeeds with greater frequency and yields higher expected value (expected cash flow)

$$CF_{l,s} > CF_{h,s} \quad \textit{and} \quad \alpha_h > \alpha_l \quad \textit{such that} \quad EV_h > EV_l$$

Expected cash flow: $EV_h = \alpha_h\, CF_{h,s} + (1 - \alpha_h)0$ **>** $\alpha_l\, CF_{l,s} + (1 - \alpha_l)0 = EV_l$

Problem statement: the bank wants to avoid that, after signing the contract fixing the risk-adjusted interest rate r_L , the borrower who has promised to carry out the safe project *h* invests money L on the risky *l-project*

Expected profit

$$E\Pi_h = EV_h - \alpha_h(1 + r_L)L = \alpha_h CF_{s,h} - \alpha_h(1 + r_L)L$$

$$E\Pi_l = EV_l - \alpha_l(1 + r_L)L = \alpha_l CF_{s,l} - \alpha_l(1 + r_L)L$$

Now consider a type *h* project submitted to the bank for the purpose of contracting a money-lone of size L. To avoid post-contractual opportunism the contract must fulfill the

Borrower's incentive compatibility constraint:

$E\Pi_h \geq E\Pi_l$ that is: $EV_h - \alpha_h(1 + r_L)L \geq EV_l - \alpha_l(1 + r_L)L$

Can be written as: $EV_h - EV_l \geq (\alpha_h - \alpha_l)(1 + r_L)L$

There exists a maximum incentive-compatible interest factor $(1 + r_{Lmax})$ such that the borrower prefers safe project h to risky project l

$$(1 + r_{Lmax}) = \frac{EV_h - EV_l}{(\alpha_h - \alpha_l)L} \geq (1 + r_L)$$

That is: $r_L \leq r_{Lmax}$

Lender's participation constraint:

$$\frac{1+r}{\alpha_h} \leq 1 + r_L$$

$$\frac{1+r}{\alpha_h} - 1 \leq r_L$$

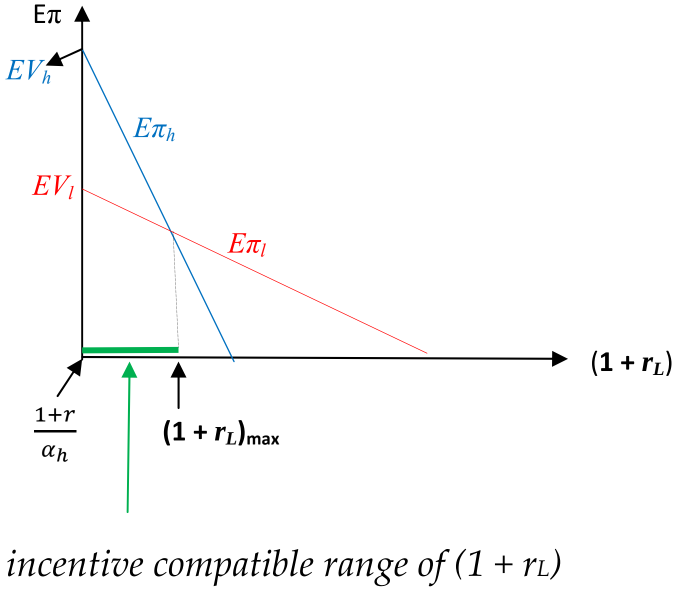
Borrower's incentive compatibility constraint:

$$r_L \leq r_{Lmax}$$

The loan contract is feasible if and only if:

$$\frac{1+r}{\alpha_h} - 1 \leq r_L \leq r_{Lmax}$$

Incentive compatible interest factor



Effect of a credit constraint D on the incentive-compatible interest rate

Credit constraint:
Suppose the bank is unwilling to finance the whole project cost L.
The borrower is forced to contributing to the project with own funds D

Expected profit with credit constraint: D > 0

$$E\Pi_h = EV_h - D - \alpha_h(1 + r_L)(L - D)$$
$$E\Pi_l = EV_l - D - \alpha_l(1 + r_L)(L - D)$$

The incentive-compatibility constraint that makes the low-risk project h more attractive than the high-risk project l is

$$E\Pi_h \geq E\Pi_l,$$

that is: $$EV_h - \alpha_h(1 + r_L)(L - D) \geq EV_l - \alpha_l(1 + r_L)(L - D)$$

$$EV_h - EV_l \ \geq (\alpha_h - \alpha_l)(1 + r_L)(L - D)$$

This can be written as:

$$D(\alpha_h - \alpha_l) + EV_h - EV_l \geq (\alpha_h - \alpha_l)(1 + r_L)L$$

$$\left(1 + r_{L,max} | D > 0\right) = \frac{EV_h - EV_l}{(\alpha_h - \alpha_l)(L - D)} \geq (1 + r_L)$$

The higher D, the higher the higher the maximum incentive compatible, risk adjusted interest rate $r_{L,max}$

Conclusion:

In spite of risk neutrality, because the credit constraint D is now forcing the entrepreneur to take some of the losses in case of failure, the bank can fix a higher incentive-compatible interest rate r_L.

The upper-bound $r_{L,max}$ to the incentive-compatible risk-adjusted interest rate r_L is an increasing function of D. By imposing a higher credit constraint D, the bank is free to fix a higher incentive compatible interest rate than it would be the case otherwise.

Exercise: Borrower and lender are risk neutral. Project type is common information, but there are high monitoring costs that cause 'hidden action' after the contract is signed.

Two projects: A, B

Cost: $L_A = L_B = 100$

Cash flow if s: $CF_{s,A} = 600$ $CV_{S,B} = 880$ $\alpha_{A,S} = 0.8$ $\alpha_{B,S} = 0.5$

Cash flow if s: $CF_{f,A} = 0$ $CV_{f,B} = 0$

The 'safe' interest rate is r = 0.05

What is the maximum incentive compatible interest rate r_L?

Is the bank prepared to lend money L at such a rate?

Solution:

$$EV_A = 600 * 0.8 = 480$$

$$EV_B = 880 * 0.5 = 440$$

$r_{L,\,max}$ *is the max. value of* r_L *such that:*

$E\Pi_A \geq E\Pi_B$ that is: $EV_A - \alpha_A(1 + r_L)L \geq EV_B - \alpha_B(1 + r_L)L$

Can be written as: $EV_A - EV_B \geq (\alpha_A - \alpha_B)(1 + r_L)L$

This implies $(1 + r_{Lmax}) = \frac{EV_A - EV_B}{(\alpha_A - \alpha_B)L} \geq (1 + r_L)$

$$(1 + r_{Lmax}) = \frac{40}{0.3 * L} = 1.33$$

That is, the **maximum incentive compatible interest rate** is r_{Lmax}= 0.33

But, is the bank prepared to lend at an interest rate $r_L \leq 0.33$?
Too see this, we must check the bank's **participation constraint**:
we call $r_{L,A}$, $r_{L,B}$ the **lowest interest rate at which the bank is prepared to lend money L on projects A, B, respectively.**

Participation constraint:

$\alpha_A(1 + r_{L,A})L = (1 + r)L$ $\mathbf{1 + r_{L,A} = \frac{1+r}{\alpha_A} = \frac{1.05}{0.8} = 1.3125}$

Since at the risk interest rate $r_{L,max}$ borrower's participation constraint is satisfied ($E\Pi_A > 0$), we can conclude:

Answer : The loan contract L financing type A project is feasible if :

$$\mathbf{1.3125 \leq 1 + r_L \leq 1.33}$$

Exercise: Same data as above, except that

The safe interest rate r = 0.1.

Lender's participation constraint is then $\mathbf{1+r_{L,A} \geq \frac{1+r}{\alpha_A} = \frac{1.1}{0.8} = 1.375}$

Now the loan contract L_A is not feasible (the loan contract L_A = 100 is not signed) because:

Participation-constrained interest $\boldsymbol{r_{L,A}}$ = 0.375 > 0.333 **=** $\boldsymbol{r_{L,max}}$ = max incentive-compatible interest rate.

.

The bank will then propose a different loan contract, such that the borrower has to participate with wealth D to project finance, and the bank contributes $L - D$ = size of the loan.

This will raise $\boldsymbol{r_{L,max}}$ according to:

$$\left(1 + r_{L,max}\right) = \frac{EV_A - EV_B}{(\alpha_A - \alpha_B)(L-D)} = \frac{40}{0.3(100-D)}$$

If D = 20, then $r_{L,max}$ = 0.666 > $\boldsymbol{r_{L,A}}$ **and at** $\boldsymbol{r_{L,A}}$ **<** $\boldsymbol{r_L} < r_{L,max}$ **the contract meets the participation and the incentive-compatibility constraints.**