Written test of 03 06 2020: solution

A. *A price taking agent with utility* $\boldsymbol{U = x_1^{1/2} x_2^{1/2}}$ *and money income m = 180, faces money prices* $p_1 = 1$, $p_2 = 1$. *Discuss the substitution effect* Δx_1^s *produced by* $\Delta p_1 = +2$

The agent has Cobb-Douglas preferences, with $MRS = \frac{MU_1}{MU_2} = \frac{1/2}{1/2}\frac{x_2}{x_1} = \frac{x_2}{x_1}$

With such preferences the demand for good 1 is $x_1 = \frac{1}{2}\frac{m}{p_1}$.

Thus, at initial prices $p_1 = 1$, $p_2 = 1$ we have $\bar{x}_1 = \frac{1}{2}\frac{180}{p_1} = 90$, while at the new prices $p'_1 = 3$, $p_2 = 1$ *we have* $x_1 = \frac{1}{2}\frac{180}{p'_1} = 30$. The price change $\Delta p_1 = p'_1 - p_1 = +2$ is producing the demand change $\Delta x_1 = x_1 - \bar{x}_1 = -60$.

The change in demand Δx_1 can be decomposed as $\Delta x_1 = \Delta x_1^m + \Delta x_1^s$. Slutsky's income effect Δx_1^m is that part of the change in demand caused by the change of purchasing power associated with the price change $\Delta p_1 = +2$. Slutsky's substitution effect Δx_1^s is that part of the change in demand caused by the price change $\Delta p_1 = +2$, *leaving purchasing power unchanged*. To measure Δx_1^s we must first compute the 'compensated income' *m'* that leaves purchasing power unchanged at $p'_1 = 3$, as it was at $p_1 = 1$.

$m' = m + \bar{x}_1 \Delta p_1 = 180 + 90 \cdot 2 = 360$

At compensated income *m'*, the compensated demand for good 1 is

$$x_1^s = \frac{1}{2}\frac{m'}{p'_1} = \frac{360}{6} = 60$$

Slutsky's substitution effect is then $\Delta x_1^s = x_1^s - \bar{x}_1 = 60 - 90 = -30$.

Slutsky's income effect is $\Delta x_1^m = \Delta x_1 - \Delta x_1^s = -60 - (-30) = -30$

B. *Agent A has utility $U(W)=W^{1/2}$. Her endowment is a risky asset with money value $W_2 = 400$, in state 2, and $W_1 = 0$ in state 1. States 1 and 2 occur with probability 1/3, 2/3 respectively. Discuss A's marginal rate of substitution, attitude towards risk, and optimum insurance K, if A can ensure wealth at premium* $\gamma = 1/2$.

A's expected utility is $EU(W) = \frac{1}{3}U(W_1) + \frac{2}{3}U(W_2)$ and her marginal rate of substitution is:

$$\boldsymbol{MRS = -\frac{\frac{dEU}{dW_1}}{\frac{dEU}{dW_2}} = -\frac{MU_{W_1}}{MU_{W_2}} = -\frac{\frac{1}{3}W_1^{-\frac{1}{2}}}{\frac{2}{3}W_2^{-\frac{1}{2}}} = -\frac{1}{2}\left(\frac{W_2}{W_1}\right)^{1/2}}$$

$\boldsymbol{MRS}$ is the slope of indifference curves in the space of contingent wealth (W_1, W_2). Since $|\boldsymbol{MRS}|$ is decreasing with respect to W_1, this means that indifference curves are *strictly convex* in the space (W_1, W_2), and the consumer is *risk averse*. This is also evident from the fact that consumer's marginal utility for sure wealth $\frac{dU(W)}{dW} = \frac{1}{2}W^{-1/2}$ is a decreasing function of W.
The insurance premium $\gamma = 1/2$ is higher than the probability 1/3 of damage D = 400. Since the consumer is risk averse, she will buy less the full insurance, that is, K < D.

To determine the numerical value of K we proceed as follows.
The first order condition for optimum insurance is:

$$|MRS| = \boldsymbol{\frac{1}{2}\left(\frac{C_2}{C_1}\right)^{1/2} = \frac{\gamma}{1-\gamma} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1}$$

where $\boldsymbol{C_1}$ is consumption in state 1 (bad state) and $\boldsymbol{C_2}$ is consumption in state 2 (good state). The above condition yields

$\left(\frac{C_2}{C_1}\right) = \mathbf{4,}$ or $C_2 = 4C_1$

We also have:

$$C_1 = K - \gamma K = \frac{1}{2}K$$

$$C_2 = W_2 - \gamma K = 400 - \frac{1}{2}K$$

$$C_2 = 400 - \frac{1}{2}K = 4C_1 = 2K$$

$$2K + \frac{1}{2}K = \frac{5}{2}K = 400$$

Optimum insurance is then $K^* = 160$

C. *In a competitive industry every firm has production function* $y = x_1^{1/3} x_2^{2/3}$ *Factor prices are* w_1*=4,* w_2*=1. Determine firms' cost function and the long-run market price*

The cost function $C(y)$ defines the *minimum* cost at which the firm can produce the output y at the given factor prices. $C(y) = \boldsymbol{w_1 x_1 + w_2 x_2}$ such that $y = x_1^{1/3} x_2^{2/3}$ and the inputs $\boldsymbol{x_1, x_2}$ are cost-minimising at factor prices $\boldsymbol{w_1, w_2}$.
To find $C(y)$ we must first consider the cost-minimisation problem of our competitive firm. The first order condition for cost-minimisation is:

$$\boldsymbol{TRS = \frac{MP_1}{MP_2} = \frac{w_1}{w_2}}$$

Using firm's production function and the given factor prices we compute:

$$\boldsymbol{TRS = \frac{MP_1}{MP_2} = \frac{1/3}{2/3}\frac{x_2}{x_1} = \frac{w_1}{w_2} = 4}$$

This yields $\frac{x_2}{x_1} = \boldsymbol{8}$, that is, $\boldsymbol{x_2 = 8x_1}$. We can now use the production function to write:

$$y = x_1^{1/3} x_2^{2/3} = x_1^{1/3}(8x_1)^{2/3} = 8^{2/3}x_1 = 4x_1$$

$$x_1 = \frac{1}{4}y \quad \text{and} \quad x_2 = 8x_1 = 2y$$

$$C(y) = \boldsymbol{w_1 x_1 + w_2 x_2} = \boldsymbol{4} \cdot \frac{1}{4}y + 1 \cdot 2y = 3y$$

The average and marginal cost is $AC = \frac{C(y)}{y} = 3$. Notice that the average cost is constant, consistently with the constant-returns to scale production function. Thus, $AC = AC_{min} = 3$.
If the market price p is higher than the average cost, every firm in the industry is earning a positive profit; this profit opportunity provides an incentive to the entry of new firms into the industry, leading to a higher industry output and to a lower market price. In this way, the free entry of firms in a competitive industry drives the long-run market price towards the minimum average cost. The long-run market price is then $p = AC_{min} = 3$.

D. *What is a 'public good'? An economy consists of two agents A, B spending their money endowment* $m_A = m_B = 600$ *to buy a public good G and a private good x. Their preferences are described by* $U(G, x) = 20logG + x$*. If the market prices of G and x are* $p_G = 1$*,* $p_x = 1$*, determine the Pareto efficient quantity G*.*

A public good G is defined by the properties of non-rivalry and non-excludability. Non-rivalry in consumption means that consumption of G by one agent does lower the quantity of G available for the other agents. Non-excludability means that, as long as there is a positive provision G of the public good in the economy, every agent has access to that quantity G.

The conditions for a Pareto efficient provision G* of a public good in the economy differ drastically from those pertaining to private goods. When goods are private, a necessary condition for a Pareto efficient allocation is that agents do not have incentives to engage in trade, because the mutually profitable trade opportunities are exhausted. For instance, in a pure exchange economy with two agents A and B, this requires that the condition $|MRS^A| = |MRS^B|$ is fulfilled. Thus, in a Pareto efficient allocation of an economy with private goods economic agents have the same marginal rate of substitution between any two goods.

This is not the case in an economy with public goods. In the economy considered in the exercise above, the Pareto efficient provision G* of the public good requires:

$$|MRS^A| + |MRS^B| = \frac{p_G}{p_x}$$

where $|MRS^A| = \frac{\frac{dU^A(G,x)}{dG}}{\frac{dU^A(G,x)}{dx}}$ and $|MRS^B| = \frac{\frac{dU^B(G,x)}{dG}}{\frac{dU^B(G,x)}{dx}}$

Since agents A and B have identical preferences, we can write:

$$|MRS^A| = |MRS^B| = \frac{20}{G}$$

and the condition for Pareto efficiency is:

$$|MRS^A| + |MRS^B| = \frac{20}{G} + \frac{20}{G} = \frac{p_G}{p_x} = 1$$

This yields $G^* = 40$