Microeconomics Written Test 04 06 2020: solution

A. *A price taking agent with utility* $U = x_1^{1/3} x_2^{2/3}$ *and money income m = 180, faces prices* $p_1 = 2$*,* $p_2 = 2$*. What part of the demand change* Δx_1 *caused by* $\Delta p_1 = -1$ *is explained by the income effect* Δx_1^m*?*

The agent has Cobb-Douglas preferences, with $MRS = \frac{MU_1}{MU_2} = \frac{1/3}{2/3}\frac{x_2}{x_1} = \frac{1}{2}\frac{x_2}{x_1}$

With such preferences the demand for good 1 is $x_1 = \frac{1}{3}\frac{m}{p_1}$.

Thus, at initial prices $p_1 = 2$, $p_2 = 2$ we have $\bar{x}_1 = \frac{1}{3}\frac{180}{p_1} = 30$, while at the new prices $p'_1 = 1$, $p_2 = 1$ *we have* $x_1 = \frac{1}{3}\frac{180}{p'_1} = 60$. The price change $\Delta p_1 = p'_1 - p_1 = -1$ is producing the demand change $\Delta x_1 = x_1 - \bar{x}_1 = +30.$

The change in demand Δx_1 can be decomposed as $\Delta x_1 = \Delta x_1^m + \Delta x_1^s$. Slutsky's income effect Δx_1^m is that part of the change in demand caused by the change of purchasing power associated with the price change $\Delta p_1 = -1$. Slutsky's substitution effect Δx_1^s is that part of the change in demand caused by the price change Δp_1 *while leaving purchasing power unchanged*. To measure Δx_1^s we must first compute the *compensated income m'* that leaves purchasing power unchanged at $p'_1 = 1$, as it was at $p_1 = 2$.

$m' = m + \bar{x}_1 \Delta p_1 = 180 + 30 \cdot (-1) = 150$

At compensated income *m' = 150,* the compensated demand for good 1 is

$$x_1^s = \frac{1}{3}\frac{m'}{p'_1} = \frac{150}{3} = 50$$

Slutsky's substitution effect is then $\Delta x_1^s = x_1^s - \bar{x}_1 = 50 - 30 = +20.$

Slutsky's income effect is $\Delta x_1^m = \Delta x_1 - \Delta x_1^s = 30 - 20 = +10$

B. *Agent A wealth is a risky asset with value W = 200 in state 2 and W = 0 in state 1; her utility is* $U(W) = 4 + 2W$. *State 1 and 2 occur with probability 1/3, 2/3 respectively. Discuss A's marginal rate of substitution, attitude towards risk, and optimum insurance K, if A can ensure wealth at premium γ = 1/2.*

A's expected utility is $EU(W) = \frac{1}{3}U(W_1) + \frac{2}{3}U(W_2)$ and her marginal rate of substitution is:

$$\boldsymbol{MRS} = -\frac{\frac{dEU}{dW_1}}{\frac{dEU}{dW_2}} = -\frac{MU_{W_1}}{MU_{W_2}} = -\frac{\frac{1}{3}}{\frac{2}{3}} \cdot \frac{2}{2} = -\frac{1}{2}$$

$\boldsymbol{MRS}$ is the slope of indifference curves in the space of contingent wealth (W_1, W_2). Since the agent has a linear utility function for sure wealth W, the marginal utility of sure wealth is constant, hence her $|\boldsymbol{MRS}|$ is also constant and equal to the ratio between the probability of state 1 and the probability of state 2. This means that indifference curves are *straight lines* in the space (W_1, W_2) with slope $\boldsymbol{MRS} = -\frac{1}{2}$ and the consumer is *risk neutral*.
The insurance premium $\gamma = 1/2$ is higher than the probability 1/3 of damage D = 200. Since the consumer is risk neutral, she will not buy any insurance, that is, K = 0.
In fact, the risk neutral agent would be just indifferent between buying and not buying insurance, if the premium γ was fair ($\gamma = 1/3$), in which case we would have

$$|MRS| = \frac{1}{2} = \frac{\gamma}{1-\gamma} \text{ at } \boldsymbol{\gamma} = \frac{1}{3}$$

With an insurance premium which is higher than 1/3, the risk neutral agent will therefore decide to buy zero insurance.

C. *In a competitive industry every firm has production function* $y = x_1^{1/2} x_2^{1/2}$*; factor prices are* w_1*=2,* w_2*=8. Determine firms' cost function and the long-run market price.*

The cost function $C(y)$ defines the *minimum* cost at which the firm can produce the output y at the given factor prices. $C(y) = \boldsymbol{w_1 x_1} + \boldsymbol{w_2 x_2}$ such that $y = x_1^{1/2} x_2^{1/2}$ and the inputs $\boldsymbol{x_1, x_2}$ are cost-minimising at factor prices $\boldsymbol{w_1, w_2}$.
To find $C(y)$ we must first consider the cost-minimisation problem of our competitive firm. The first order condition for cost-minimisation is:

$$\boldsymbol{TRS = \frac{MP_1}{MP_2} = \frac{w_1}{w_2}}$$

Using firm's production function and the given factor prices we compute:

$$\boldsymbol{TRS = \frac{MP_1}{MP_2} = \frac{1/2\, x_2}{1/2\, x_1} = \frac{x_2}{x_1} = \frac{w_1}{w_2} = \frac{1}{4}}$$

This yields $\frac{x_2}{x_1} = \frac{1}{4}$, that is, $\boldsymbol{x_1 = 4x_2}$. We can now use the production function to write:

$$y = x_1^{1/2} x_2^{1/2} = (4x_2)^{1/2} x_2^{1/2} = 4^{1/2} x_2 = 2x_2$$

$$x_2 = \frac{1}{2} y \qquad \text{and} \qquad x_1 = 4x_2 = 2y$$

$$C(y) = w_1 x_1 + w_2 x_2 = 2 \cdot 2y + 8 \cdot \frac{1}{2} y = 8y$$

The average and marginal cost is $AC = \frac{C(y)}{y} = 8$. Notice that the average cost is constant, consistently with the constant-returns to scale production function. Thus, $AC = AC_{min} = 8$.
If the market price p is higher than the average cost, every firm in the industry is earning a positive profit; this profit opportunity provides an incentive to the entry of new firms into the industry, leading to a higher industry output and to a lower market price. In this way, the free entry of firms in a competitive industry drives the long-run market price towards the minimum average cost. The long-run market price is then $p = AC_{min} = 8$.

D. *Explain why: (1) a competitive-equilibrium of a pure-exchange economy is not Pareto efficient only if there is a consumption externality E; (2) Pareto efficiency is re-established by the definition of property-rights on the externality E. (3) the Pareto efficient quantity E* may not depend on the allocation of property rights.*

(1) In the absence of externalities, decentralized decision making by price-taking agents makes the competitive equilibrium of a pure exchange economy Pareto efficient. This follows from the 1st theorem of welfare economics. Intuitively, the equilibrium allocation is by definition feasible, and since the agents take utility maximising decisions in the face of the same market prices, in equilibrium (assuming for simplicity that they consume a positive quantity of each good) they have the same MRS between any pair of goods. This means that the mutually profitable trade opportunities are exhausted in equilibrium.

(2) The argument above fails in the presence of a consumption externality E. This is because it is in the nature of decentralized decision making *in the presence of self-regarding preferences*, that every agent disregards the effect of his/her decisions on the welfare of the others. It follows that in a competitive equilibrium some agent *j* may be willing to pay some other agent i, in order to modify his/her consumption. The competitive equilibrium fails to be efficient precisely because the market for the externality E is missing, as a result of the ill-defined property rights on the externality. *The definition of such property rights by a public authority, creates the market for E that was previously missing.* Decentralised decision making will then lead to a Pareto efficient equilibrium characterised by the property that every agent has the same MRS between E and every other good.

(3) In general, the individual demand for E will depend, not only on prices, but also on agent's wealth. For this reason, the equilibrium and Pareto efficient quantity of E depends on the allocation of property rights, to the extent that they affect agent's wealth. In the special case in which individual utility is quasi-linear with the form $U(E, x) = v(E) + x$ with $v'^{(E)} > 0$, $v''^{(E)} < 0$, the individual demand for E does not depend on agent's wealth, but depends only on prices. In this particular case, the Pareto efficient quantity of E does not depend on the allocation of property rights (*Coase Theorem*).