Solution to the written test of 02/09/2020

A.

Determine the optimum choice at prices $p_1 = 1$, $p_2 = 2$ of a price-taking agent with utility $U = x_1^{1/2} x_2^{1/2}$ and money income $m = 240$. Determine Slutsky's substitution effect Δx_1^s produced by $\Delta p_1 = +1$, that is $p_1' = 2$.

The agent has Cobb-Douglas preferences, with $MRS = \frac{MU_1}{MU_2} = \frac{1/2}{1/2}\frac{x_2}{x_1} = \frac{x_2}{x_1}$

With such preferences the demand for good 1 is $x_1 = \frac{1}{2}\frac{m}{p_1}$.

Thus, at initial prices $p_1 = 1$, $p_2 = 1$ we have $\bar{x}_1 = \frac{1}{2}\frac{240}{p_1} = 120$, while at the new prices $p'_1 = 2$, $p_2 = 2$ *we have* $x_1 = \frac{1}{2}\frac{240}{p'_1} = 60$. The price change $\Delta p_1 = p'_1 - p_1 = +1$ is producing the demand change $\Delta x_1 = x_1 - \bar{x}_1 = -60$.

The change in demand Δx_1 can be decomposed as $\Delta x_1 = \Delta x_1^m + \Delta x_1^s$. Slutsky's income effect Δx_1^m is that part of the change in demand caused by the change of purchasing power associated with the price change $\Delta p_1 = +1$. Slutsky's substitution effect Δx_1^s is that part of the change in demand caused by the price change $\Delta p_1 = +1$, *leaving purchasing power unchanged*. To measure Δx_1^s we must first compute the 'compensated income' m' that leaves purchasing power unchanged at $p'_1 = 2$, as it was at $p_1 = 1$.

$$m' = m + \bar{x}_1 \Delta p_1 = 240 + 120 \cdot 1 = 360$$

At compensated income m', the compensated demand for good 1 is

$$x_1^s = \frac{1}{2}\frac{m'}{p'_1} = \frac{360}{4} = 90$$

Slutsky's substitution effect is then $\Delta x_1^s = x_1^s - \bar{x}_1 = 90 - 120 = -30$.

Slutsky's income effect is $\Delta x_1^m = \Delta x_1 - \Delta x_1^s = -60 - (-30) = -30$

B.

Agent A has utility *U(W)=W^{1/2}*. Her wealth is a risky asset B with money value W_1 = *64* in state 1 and W_2 = *100* in state 2. States 1 and 2 occur with probability *1/2, 1/2*. Discuss A's *expected utility, marginal rate of substitution*, and the *certainty equivalent* of B.

A's expected utility is $EU(W) = \frac{1}{2}U(W_1) + \frac{1}{2}U(W_2) = \frac{1}{2}8 + \frac{1}{2}10 = 9$.
Her marginal rate of substitution is:

$$\boldsymbol{MRS = -\frac{\frac{dEU}{dW_1}}{\frac{dEU}{dW_2}} = -\frac{MU_{W_1}}{MU_{W_2}} = -\frac{\frac{1}{2}W_1^{-\frac{1}{2}}}{\frac{1}{2}W_2^{-\frac{1}{2}}} = -\left(\frac{W_2}{W_1}\right)^{\frac{1}{2}} = -\frac{5}{4}}$$

$\boldsymbol{MRS}$ is the slope of indifference curves in the space of contingent wealth (W_1, W_2). Since $|\boldsymbol{MRS}|$ is decreasing with respect to W_1, this means that indifference curves are *strictly convex* in the space (W_1, W_2), and the consumer is *risk averse*. This is also evident from the fact that consumer's marginal utility for sure wealth $\frac{dU(W)}{dW} = \frac{1}{2}W^{-1/2}$ is a decreasing function of W.

The certainty equivalent of W is the amount of sure money x yielding a utility $U(x)$ equal to the expected utility of W. Thus $U(x) = x^{1/2} = EU(W) = 9$. Thus $x = 81$ and since the consumer is risk-averse, the certainty equivalent of W is lower than the expected value of W, that is:

$$CE(W) = x = 81 < E(W) = \frac{1}{2}64 + \frac{1}{2}100 = 82$$

C. What is a Cournot duopoly? Determine the Cournot market price p and the industry profit π if inverse market demand is $p = 120 - y$ and each firm has cost function $C = 800$.

A Cournot duopoly is a market form identified by an industry with two firms taking simultaneous decisions about their respective output, based on the expectation concerning the quantity that the other firm is about to produce. The behaviour of the two firms is thus described by the reaction functions $y_1 = f(y_2^e)$ and $y_2 = f(y_1^e)$. A Cournot equilibrium is an output pair y_1^*, y_2^* such that $y_1^* = f(y_2^*)$ and $y_2^* = f(y_1^*)$. In other words, expectations are fulfilled and each firm takes a best response to the other firm decision. Notice that this defines a Nash equilibrium in the language of non-cooperative games.

Since $y = y_1 + y_2$ we can write $\boldsymbol{p} = \mathbf{120} - y_1 - y_2$ and the revenue of firm 1 is $R_1 = py_1 = (\mathbf{120} - y_1 - y_2)y_1 = 120y_1 - y_1^2 - y_2y_1$.

The marginal revenue of firm 1 is then $MR_1 = 120 - 2y_1 - y_2$ while the marginal cost $\frac{\partial C}{\partial y_1} = 0$.

First-order condition for firm's profit maximisation is marginal revenue equals marginal cost, to the effect that $MR_1 = 120 - 2y_1 - y_2 = 0$, that is $y_1 = \frac{1}{2}(120 - y_2)$, $y_1 = 60 - \frac{1}{2}y_2$.

Since the two firms are identical, there exists a symmetric Cournot equilibrium with $y_1 = y_2$ and we can write $y_1 = 60 - \frac{1}{2}y_1$, or $\frac{3}{2}y_1 = 60$, $y_1 = 40 = y_2$, $y = 80$, $p = 120 - y = 40$.

Firm's revenue is $py_1 = py_2 = 40 \cdot 40 = 1600$ and firm's profit is $\pi_1 = \pi_2 = 1600 - 800 = 800$. Industry profit is $\pi = \pi_1 + \pi_2 = 1600$.

D

In a competitive industry every firm has production function $y = x_1 + 4x_2$ Factor prices are *w_1= 4, w_2* = 12. Determine firms' cost function and the long-run market price.

The cost function $C(y)$ defines the *minimum* cost at which the firm can produce the output y at the given factor prices. $C(y) = \boldsymbol{w_1 x_1} + \boldsymbol{w_2 x_2}$ such that $y = x_1 + x_2$ and the inputs $\boldsymbol{x_1}, \boldsymbol{x_2}$ are cost-minimising at factor prices $\boldsymbol{w_1}, \boldsymbol{w_2}$.

To find $C(y)$ we must first consider the cost-minimisation problem of our competitive firm. The first order condition for an interior solution to firm's cost-minimisation problem is:

$$\boldsymbol{TRS} = \frac{\boldsymbol{MP_1}}{\boldsymbol{MP_2}} = \frac{\boldsymbol{w_1}}{\boldsymbol{w_2}}$$

In the resent case however the solution is not interior, since

$$\boldsymbol{TRS} = \frac{\boldsymbol{MP_1}}{\boldsymbol{MP_2}} = \frac{\mathbf{1}}{\mathbf{4}} < \frac{\boldsymbol{w_1}}{\boldsymbol{w_2}} = \frac{\mathbf{1}}{\mathbf{3}}$$

This implies that the firm will use only factor 2 at the given prices, to the effect that $x_1 = 0$ and $y = 4x_2$, that is $x_2 = \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{y}$. We can write:

$$C(y) = \boldsymbol{w_2 x_2} = \mathbf{12} \cdot \frac{1}{4} y = 3y$$

The average and marginal cost is $AC = \frac{C(y)}{y} = 3$. Notice that the average cost is constant, consistently with the constant-returns to scale production function. Thus, $AC = AC_{min} = 3$.

If the market price p is higher than the average cost, every firm in the industry is earning a positive profit; this profit opportunity provides an incentive to the entry of new firms into the industry, leading to a higher industry output and to a lower market price. In this way, the free entry of firms in a competitive industry drives the long-run market price towards the minimum average cost. The long-run market price is then $p = AC_{min} = 3$.

E

Explain why a *consumption externality* implies that competitive market outcomes are not Pareto efficient and why Pareto efficiency may be restored by an appropriate definition of property rights.

In the absence of externalities, decentralized decision making by price-taking agents makes the competitive equilibrium of a pure exchange economy Pareto efficient. This follows from the 1st theorem of welfare economics. Intuitively, the equilibrium allocation is by definition feasible, and since the agents take utility maximising decisions in the face of the same market prices, the mutually profitable trade opportunities are exhausted in equilibrium.

The argument above fails in the presence of a consumption externality E. This is because it is in the nature of decentralized decision making *in the presence of self-regarding preferences*, that every agent disregards the effect of his/her decisions on the welfare of the others. It follows that in a competitive equilibrium some agent j may be willing to pay some other agent i, in order to modify his/her consumption. The competitive equilibrium fails to be efficient precisely because the market for the externality E is missing, as a result of the ill-defined property rights on the externality. *The definition of such property rights by a public authority, creates the market for E that was previously missing.* Decentralised decision making will then lead to a Pareto efficient equilibrium, to the extent that trade in the rights to E is costless (transaction costs are ignored).

In general, the individual demand for E (E-rights) will depend, not only on prices, but also on agent's wealth. For this reason, the equilibrium and Pareto efficient quantity of E depends on the allocation of property rights, to the extent that they affect agent's wealth. In the special case in which individual utility is quasi-linear with the form $U(E, x) = v(E) + x$ with $v'^{(E)} > 0$, $v''^{(E)} < 0$, the individual demand for E does not depend on agent's wealth, but depends only on prices. In this particular case, the Pareto efficient quantity of E does not depend on the allocation of property rights (*Coase Theorem*).