MACROECONOMICS 2016

Problem set n°1

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1) Show that the expression of the adaptive expectation $p_t^e = \lambda \sum_{i=1}^{\infty}(1-\lambda)^{i-1}P_{t-i}$ when prices are cyclical of order two, that is

$$p_t = \begin{cases} p^* \ if \ t \ is \ even \\ p^{**} \ if \ t \ is \ odd \end{cases}$$

Is given by $p_t^e = \frac{p^{**}+p^*(1-\lambda)}{2-\lambda}$ considering time $t-1$, when the expectation is formed, as odd $p^{**} > p^*$ and $0 < \lambda < 1$. What happens when λ goes to 0 and to 1?

2) Show that the expression of the adaptive expectation $p_t^e = \lambda \sum_{i=1}^{\infty}(1-\lambda)^{i-1}p_{t-i}$ when prices change suddenly, that is

$$p_t = \begin{cases} p^* & if \ t \leq T \\ p^{**} & if \ t > T \end{cases}$$

Is given by

$$p_t^e = \begin{cases} p^* & if \ t \leq T+1 \\ p^{**}[1-(1-\lambda)^{t-T-1}]+p^{**}(1-\lambda)^{t-T-1} & if \ t > T+1 \end{cases}$$

with $0 < \lambda < 1$ and $p^{**} > p^*$. What happens when t goes to infinity?

3) Show that if y_t^* and $\overline{y}_t$ are two solutions of the equation

$$y_t = aE(y_{t+1}|I_t) + b$$

then also $\underline{y_t} = \theta y_t^* + (1-\theta)\overline{y}_t$ with $0 < \theta < 1$ is also a solution of the equation.

4) Consider the (logarithmic version) of the Cagan demand function seen in class with rational expectations
$$m_t - p_t = -\alpha(E(p_{t+1}|I_t) - p_t)$$

Find the fundamental solution for p_t and compare it with the adaptive expectations solution in an economy where m_t evolves as follows

$$m_t = \begin{cases} m_0 & if \ t \leq T \\ m_T & if \ t > T \end{cases}$$

with $m_T > m_0$. Moreover, discuss if the following stochastic process b_t, with values and probabilities given by

$$b_t = \begin{cases} \dfrac{b_{t-1}}{a^2} & with \ probability \ a \\ 0 & with \ probability \ 1-a \end{cases}$$

where $a = \frac{\alpha}{1+\alpha}$, can be a bubble process part of the model solution.

5) In the continuous time version of the Ramsey model discussed in class find the Hamiltonian, the Euler optimal conditions and discuss the stationary states and phase diagram under the following hypotheses

i) $\frac{\dot{N}_t}{N_t} = nc_t$ ii) $\frac{\dot{N}_t}{N_t} = nk_t$ iii) the discount factor $e^{-\theta c_t}$ iv) the discount factor $e^{-\theta k_t}$

assuming the rest of the model to be the same

6) In the discrete time version of the Ramsey model discussed in class find the Keynes-Ramsey conditions and discuss the stationary states when

$$i)\ population\ rate\ of\ grow \frac{N_{t+1}}{N_t} = (1 + nc_t)\ ii)\ \frac{N_{t+1}}{N_t} = (1 + nk_t)$$

iii) the discount factor $\frac{1}{1+\theta k_t}$ iv) the discount factor $\frac{1}{1+\theta c_t}$ assuming the rest of the model to be the same.