

MACROECONOMICS 2017

Problem set n° 2

Prof. Nicola Dimitri

- 1) Consider the OLG with money seen in class, where the two periods endowments of the consumption good for the representative agent are $e_1 > 0$ and $e_2 > 0$, with the population growing according to the following equation $N_t = (1 + n)^t N_0$. Suppose the good is perishable, that is the storage technology is characterized by a return rate $r = -1$.

- a) Write down the budget constraints, for the two periods, and the inter-temporal budget constraint.
- b) Discuss the dynamic stability of monetary stationary state, that is the stability of the real-money market equilibrium equation $m_{t+1} = h(m_t)$, when the utility function of the representative agent $W(c_{1t}, c_{2t+1})$ is
- i) $c_{1t} + \frac{\log c_{2t+1}}{1+\theta}$ ii) $c_{1t} + \frac{c_{2t}}{1+\theta}$

- 2) Consider the OLG without money seen in class, where the two periods endowments of the consumption good for the representative agent are $e_1 = 0$ and $e_2 = 0$. Population grows as in problem 1. Suppose the utility function of the representative agent $W(c_{1t}, c_{2t+1}) = \log c_{1t} + \frac{c_{2t+1}}{1+\theta}$ and the production function $f(k_t) = k_t^a$, with $0 < a < 1$. Find the dynamic law driving the evolution of the equilibrium per capita capital k_t , discuss its stationary states and its dynamics

3)

- A) Consider the following one variable $x_t \geq 0$, discrete time, dynamical system

$$x_t = ax_{t-1}(b - x_{t-1}) + c(b - x_{t-1}) \text{ (Generalised logistic function) with } a, b, c > 0$$

Draw a graph of the function, find the Stationary States and discuss their stability. Can the system ever exhibit cyclical dynamics?

- 4) Consider the two variable, first order linear difference equation, system

$$\begin{aligned}x_t &= 2x_{t-1} + y_{t-1} + 5 \\y_t &= 0.3y_{t-1} + 1\end{aligned}$$

Find the stationary state of the system and discuss whether or not is a saddle point.