**MACROECONOMICS 2018**

**Problem set n°1**

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1. Show that the expression of the adaptive expectation $p\_{t}^{e}= λ$ $\sum\_{i=1}^{\infty }(1-λ)^{i-1}P\_{t-i}$ when prices are cyclical of order two, that is

$$p\_{t}=\left\{\begin{array}{c}p^{\*} if t is even\\p^{\*\*} if t is odd\end{array}\right.$$

Is given by $p\_{t}^{e}=\frac{p^{\*\*}+p^{\*}(1-λ)}{2-λ}$ considering time $t-1$, when the expectation is formed, as odd $p^{\*\*}>p^{\*} and 0<λ<1$. What happens when $λ$ goes to $0$ and to $1$?

1. Show that the expression of the adaptive expectation $p\_{t}^{e}= λ$ $\sum\_{i=1}^{\infty }(1-λ)^{i-1}p\_{t-i}$ when prices change suddenly, that is

$$p\_{t}=\left\{\begin{array}{c}p^{\*} if t \leq T\\p^{\*\*} if t>T\end{array}\right.$$

Is given by

$$p\_{t}^{e}=\left\{\begin{array}{c}p^{\*} if t \leq T+1\\p^{\*\*}\left[1-\left(1-λ\right)^{t-T-1}\right]+p^{\*\*}\left(1-λ\right)^{t-T-1} if t>T+1\end{array}\right.$$

with $0<λ<1$ and $p^{\*\*}>p^{\*}$.What happens when $t$ goes to infinity?

1. Show that if $y\_{t}^{\*}$ and $\overline{y}\_{t}$ are two solutions of the equation

$$y\_{t}=aE\left(I\_{t}\right)+b$$

 then also $\overline{y\_{t}}=θy\_{t}^{\*}$ + $(1-θ)\overline{y}\_{t}$ with $0<θ<1$ is also a solution of the equation.

1. Consider the (logarithmic version) of the Cagan demand function seen in class with rational expectations

$$m\_{t}-p\_{t}=-α(E\left(I\_{t}\right)-p\_{t})$$

Find the fundamental solution for $p\_{t}$ and compare it with the adaptive expectations solution in an economy where $m\_{t}$ evolves as follows

$$m\_{t}=\left\{\begin{array}{c}m\_{0} if t\leq T\\m\_{T} if t>T\end{array}\right.$$

with $m\_{T}>m\_{0}$. Moreover, discuss if the following stochastic process $b\_{t}$, with values and probabilities given by

$$b\_{t}=\left\{\begin{array}{c}\frac{b\_{t-1}}{a^{2}} with probability a \\0 with probability 1-a\end{array}\right.$$

where $a=\frac{α}{1+α}$, can be a bubble process part of the model solution.

1. In the continuous time version of the Ramsey model discussed in class find the Hamiltonian, the Euler optimal conditions and discuss the stationary states and phase diagram under the following hypotheses

$i) \frac{\dot{N}\_{t}}{N\_{t}}=nc\_{t} ii) \frac{\dot{N}\_{t}}{N\_{t}}=nk\_{t} iii) the discount factor e^{-θc\_{t}} iv) the discount factor e^{-θk\_{t}}$ assuming the rest of the model to be the same

1. In the discrete time version of the Ramsey model discussed in class find the Keynes-Ramsey conditions and discuss the stationary states when

$$i) population rate of grow\frac{N\_{t+1}}{N\_{t}}=(1+nc\_{t}) ii) \frac{N\_{t+1}}{N\_{t}}=(1+nk\_{t})$$

$ iii) the discount factor \frac{1}{1+θk\_{t}} iv) the discount factor \frac{1}{1+θc\_{t}}$ assuming the rest of the model to be the same.