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AY 2017-2018
Economics of Money and Banking
Simulation second mid term test (Monday 18 June)
Solutions

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Available time: 90 Minutes

1. (11 points)

In the General Equilibrium with banks model discussed in class suppose consumers have the following utility function

$$U(c_1, c_2) = c_1 + log c_2$$

and that the firms production function is

$$f(I) = log(1 + I)$$

where $c_1, c_2, I \geq 0$. Find the General Equilibrium values of quantities and prices of this economy

Discussion

Because in equilibrium $r = r_B = r_L = r_D$ it is easier to start considering the firm profit as function of $I = L_f + B_f$ rather than L_f, B_f separately, since loans and bonds are perfect substitutes

$$\Pi_f(I) = p\left(log(1 + I)\right) - (1 + r)I \quad (1)$$

Maximising (1) with respect to I gives as FOC

$$\frac{p}{(1 + I)} - (1 + r) = 0 \quad (2)$$

Because the second derivative of (1) is $-\frac{p}{(1+I)^2} < 0$ solution to (2) provides a maximum for (1). Solving (2) gives the optimal investment level I^*as

$$I^* = \begin{cases} \frac{p}{1 + r} - 1 & if\ p > (1 + r) \\ 0 & otherwise \end{cases}$$

Assuming $p > 1 + r$ it follows then (1) becomes

$$\Pi_f(I^*) = p \log\left(\frac{p}{1 + r}\right) - p + (1 + r) > 0 \quad (3)$$

Likewise, defining $s = B_h + D_h$ as the household savings, and since the bank profit $\Pi_b = 0$, maximisation of the utility $U(c_1, c_2) = c_1 + log c_2$ with respect to savings, subject to the constraints $c_1 = w_1 - s$ and $pc_2 = \Pi_f + (1+r)s$, leads to the following first order conditions

$$-1 + \frac{(1+r)}{\Pi_f + (1+r)s} = 0 \quad (4)$$

Because second order conditions for a maximum are satisfied, solution s^* to (4) provides the optimal level of savings and is given by

$$s^* = \begin{cases} 1 - \dfrac{\Pi_f}{(1+r)} & if\ \Pi_f < (1+r) \\ 0 & otherwise \end{cases}$$

The *savings=investments* equilibrium condition

$$s^* = 1 - \frac{\Pi_f}{(1+r)} = \frac{p}{1+r} - 1 = I^*$$

will determine how p and r should relate. Finally, any combination of the remaining quantities $\mathrm{B}_f, \mathrm{B}_b, \mathrm{B}_h, \mathrm{L}_f, \mathrm{L}_b, \mathrm{D}_b, \mathrm{D}_f$, such that the rest of the equilibrium conditions are satisfied represents a general equilibrium of the economy.

2 (11 points) Consider the Diamond-Dybvig model for banks as liquidity providers, in case of negative shocks, discussed in class.

Suppose $u(c) = c^a$ with $0 < a < 1$. Find the autharky solution, the market solution and the Pareto Optimal solution for the investment level I and consumption levels c_1, c_2. Is the market solution Pareto Efficient?

Discussion

The autarky solution requires maximisation of the expected utility

$$\pi u(c_1) + (1-\pi)u(c_2) \quad (1)$$

with respect to I subject to the constraints $c_1 = 1 - I + lI$ and $c_2 = 1 - I + RI$. Assuming that FOC identify a maximum then the optimal level of investment I_a is given by

$$I_a = \frac{1 - k_a}{(1-l) - k_a(R-1)} \quad (2)$$

where $k_a = [\frac{\pi(1-l)}{(1-\pi)(R-1)}]^{\frac{1}{1-a}}$. When replaced in the constraints (2) typically provides c_1 and c_2 which differ from the market condition, $c_1 = 1$ and $c_2 = R$. The Pareto Optimal allocation is found maximising (1) given $c_1 = \frac{1-I}{\pi}$ and $c_2 = \frac{RI}{1-\pi}$

Therefore, the FOC with respect to I imply

$$a(\frac{1-I}{\pi})^{a-1} = Ra(\frac{RI}{1-\pi})^{a-1}$$

hence

$$I = \frac{1}{1+k_{po}}$$

where $k_{po} = \frac{\pi}{1-\pi}(\frac{1}{R})^{\frac{a}{1-a}}$, leading to consumption levels in the two periods that are typically different from the market solution.