3) $\pi(tp) = t\pi(p)$ HOMOGENEITY OF DEGREE 1

$k = 1$

$$\boxed{\left(f(tx) = t^k f(x)\right)} \quad k > 0$$

$$\Downarrow \quad t > 0$$

HOMOGENEOUS OF DEGREE k

PROOF

$py \geq py' \implies (tp)y \geq (tp)y'$

py: PROFIT MAX AT p

NEW PRICE VECTOR $\implies$ THAT y IS ALSO PROFIT MAXIMISING AT (tp) PRICE VECTOR

WITH $y' \neq y$

BUT $t(py) = t\pi(p)$

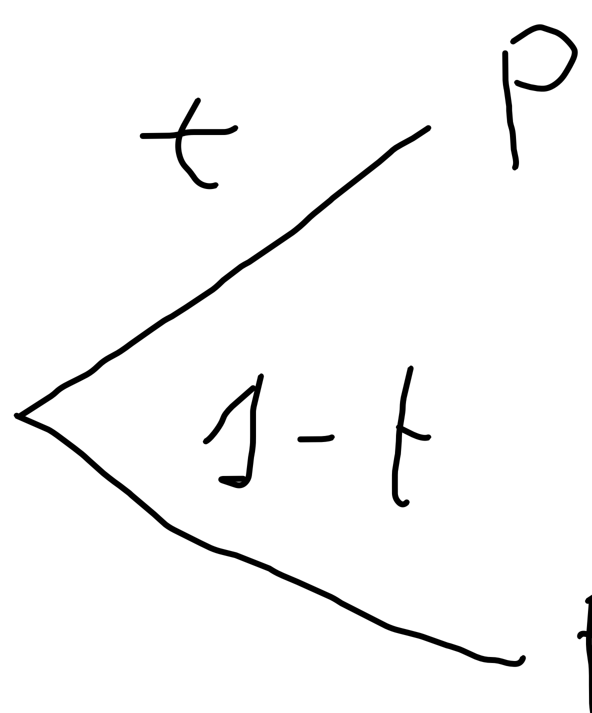
$\pi(p) \implies \pi(tp) = (tp)y = t\pi(p) = t(py)$

OBS

2) $\Pi(\underbrace{tp+(1-t)p'}_{p''}) \le \boxed{t\Pi(p)+(1-t)\Pi(p')}$ CONVEX

INTERPRETATION

SUPPOSE THE PRICE VECTOR CAN RANDOMLY OSCILLATE

a) 

$t = P$(PRICE VECTOR $= p$)

b) p'' CONSTANT

THE FIRM WILL BE BETTER OFF IN (a)

1 OUTPUT AND m INPUTS

$$\Pi(p) = p\,y(p)$$

HOTELLING'S LEMMA $y_i(p)$ SUPPLY FUNCTION OF COMMODITY i

$$\boxed{y_i(p) = \frac{\partial \Pi(p)}{\partial p_i}}$$

IF THE DERIVATIVE EXISTS

PROOF ENVELOPE THEOREM

$$\frac{\partial \Pi(p)}{\partial p_i} \overset{\downarrow}{=} \frac{\partial \Pi}{\partial p_i} = \frac{\partial (p y)}{\partial p_i} = y_i$$

Cost Function

(EXP. FUNC.) 1 OUTPUT, m INPUTS

INPUTS

$C = w\,x$
$\quad 1\times m \;\; m\times 1$

w = INPUTS PRICE VECTOR

x = INPUTS VECTOR

$$\Downarrow$$

OUTPUT LEVEL

$$C(w,y) = \min_{x} \; w\,x \quad \text{s.t. } f(x) \geq y$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \text{``} \; z$

COST FUNCTION ?

$$\Downarrow$$

$$x(w,y) \Longrightarrow C(w,y) = \underset{1\times m}{w}\,\underset{m\times 1}{x(w,y)}$$

EXAMPLES

1 OUTPUT 1 INPUT

1) $f(x) = x^a$

$$\min_{x} \underset{1\times1}{w}\,\underset{1\times1}{x} \quad \text{s.t. } f(x) = x^a \geq y$$

FACTOR DEMAND

$$x^a = y \Rightarrow \boxed{x = y^{1/a}}$$

$$\boxed{C(w,y) = w\, y^{1/a}}$$

2) $f(x) = x_1^a x_2^b$

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \quad \text{s.t. } x_1^a x_2^b \geq y$$

$\longrightarrow C(w_1, w_2, y) = K\, w_1^{a}\, w_2^{b}\, y$

OBSERVATION 1 OUTPUT m INPUTS

TO MAX PROFIT

DUALITY

TECHNOLOGICAL INFO

a) $\max_{\substack{x \\ 1\times m}} \pi = \underset{1\times 1}{p}\, \underbrace{\boxed{f(\underset{1\times m}{x})}}_{1\times 1} - \underset{1\times m}{w}\, \underset{m\times 1}{x}$

b) $\max_{\substack{y \\ 1\times 1}} \pi = \underset{1\times 1}{p}\, \underset{1\times 1}{y} - \boxed{C(w, y)}$

i) FOR EACH y WE CHOOSE THE MIN COST $C(w, y)$

ii) WE THE CHOOSE y SUCH THAT $py - C(w, y)$ IS MAX

PROPERTIES OF $C(w, y)$

1) IS NON DECREASING IN w.

NAMELY $w' \geq w \implies c(w', y) \geq c(w, y)$

2) HOMOGENOUS OF DEGREE 1 IN w

$$c(tw, y) = t\,c(w, y)$$

3) CONCAVE IN w $\big(c(tw + (1-t)w', y) \geq t\,c(w, y) + (1-t)\,c(w', y) \big)$

PROOF

SUPPOSE x IS COST MINIM. AT w, x' IS COST MIN AT w', AND x'' IS COST MIN AT $w'' = tw + (1-t)w'$

So

$$w''x'' = [\overbrace{tw + (1-t)w'}^{w''}]x'' = t w x'' + (1-t)w'x''$$

$$\geq t w x + (1-t) w' x'$$

$$\Longrightarrow c(w'', y) \geq t\, c(w, y) + (1-t)\, c(w', y)$$

4) CONTINUOUS IN w

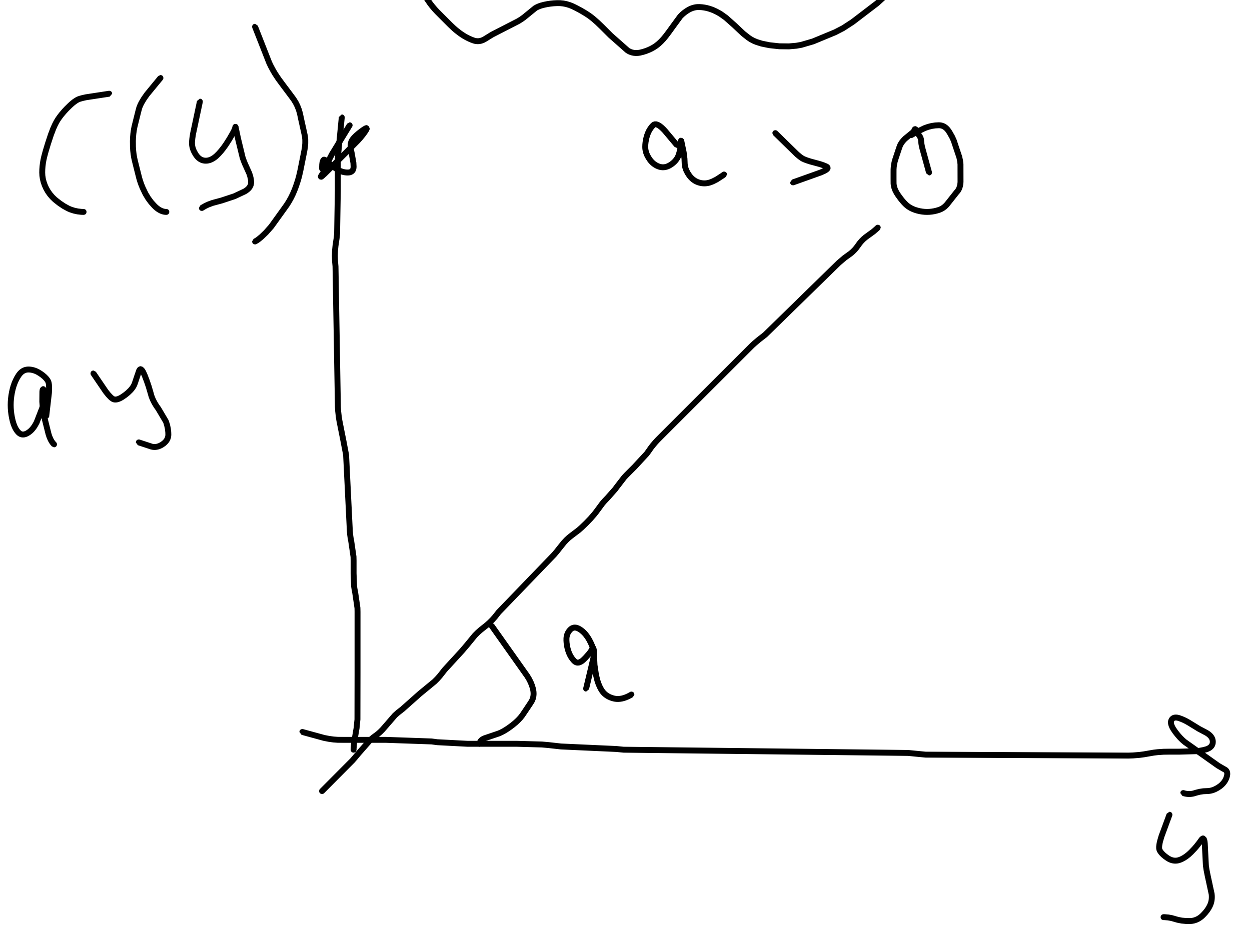
1 OUTPUT n INPUTS

PROPOSITION SUPPOSE $f(x)$ EXHIBITS CRS:

THEN $c(w,y) = y\,\underbrace{c(w,1)}$

$\Downarrow$

$c(y) = ay$



PROOF EASY

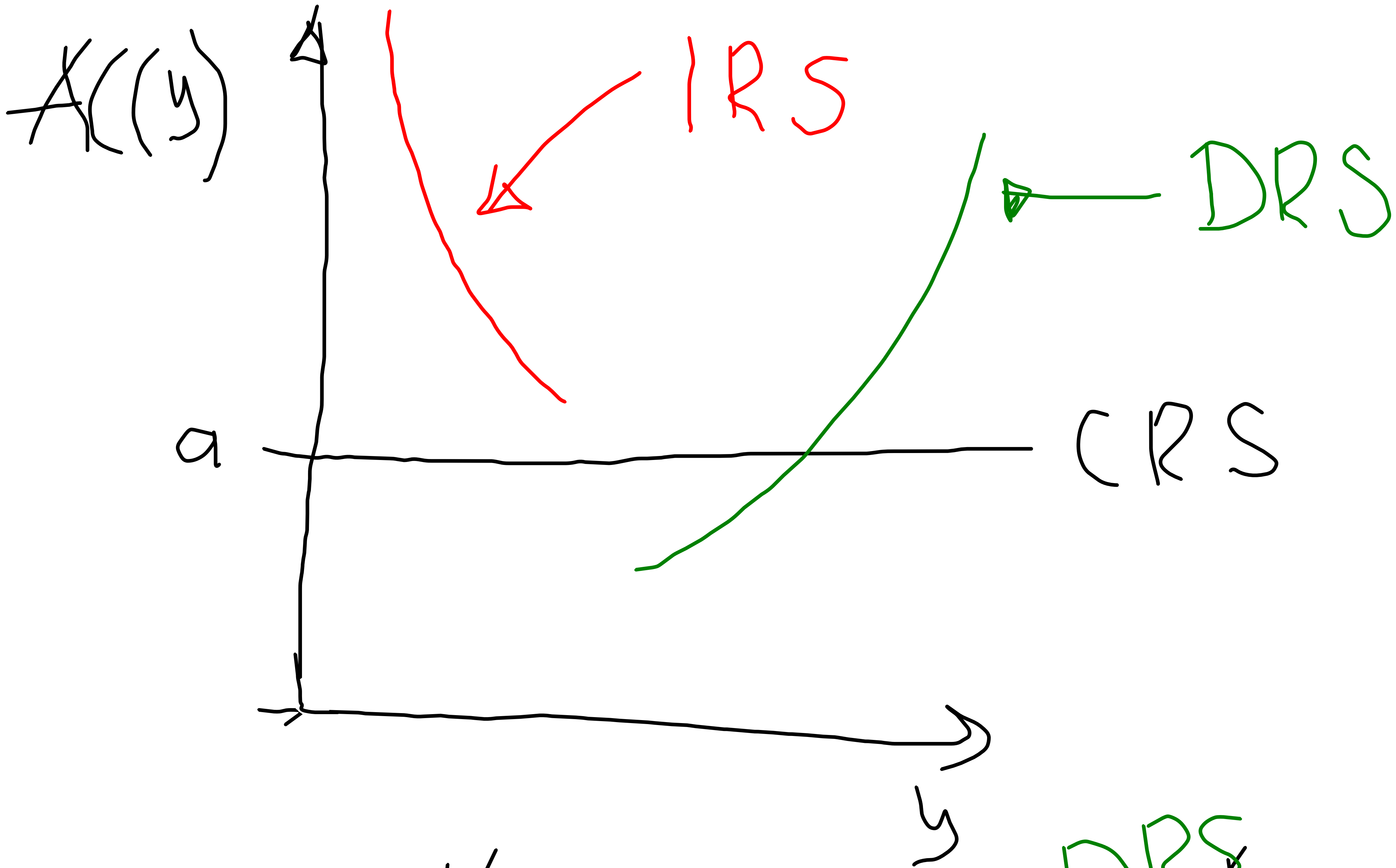
<u>SHEPARD'S LEMMA</u> IF $x_i(w,y)$ IS THE DEMAND FOR INPUT i, THEN

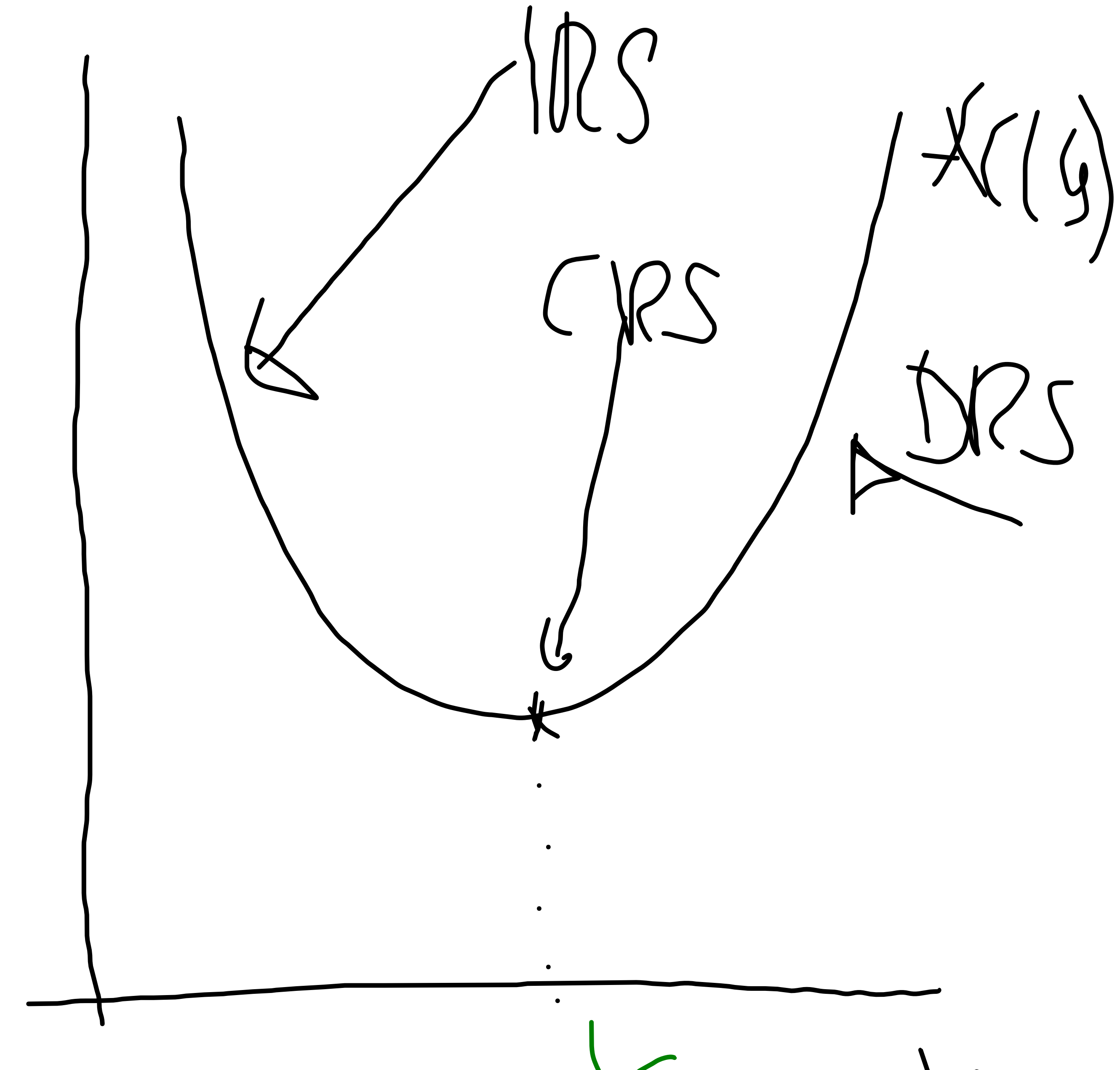
$$\boxed{x_i(w,y) = \frac{\partial C(w,y)}{\partial w_i}}$$

<u>DUALITY</u> BETWEEN PROD. FUNC. AND COST FUNCTION (1 OUTPUT)

$$C(y) = C_F + C_V(y)$$

$$\rightarrow \boxed{\frac{C(y)}{y} = \text{AVERAGE COST} = AC(y)} \qquad C'(y) = \text{MARGINAL COST} = MC(y)$$





$$\frac{C(y)}{y} = \frac{ay}{y} = a$$