ARROW - DEBREU ECONOMY
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UNCERTAINTY IN THE WALRASIAN
EQUILIBRIUM NOTION

EXAMPLE (TO INTRODUCE ARROW-DEBREU)

A-D

→ 1 GOOD (APPLES)

2 INDIVIDUALS (A, B)

2 STATES OF THE WORLD (1, 2)

} SIMPLEST A-D ECONOMY

ENDOWMENTS

$\omega_A = (\omega_{A1}, \omega_{A2}) = (1, 0)$

$\omega_B = (\omega_{B1}, \omega_{B2})$

STATE 1 STATE 2

HERE

$A \xrightarrow{\pi_A} 1 \longrightarrow 1$ APPLE

$A \xrightarrow{1-\pi_A} 2 \longrightarrow 0$ APPLE

WITH k GOOD AND m STATES EACH ENDOWMENT WILL BE A $\boxed{k \times m}$ VECTOR

FOR INSTANCE IF $k = 3$ AND $m = 3$ STATES

APPLES, PEARS, WATER

SNOW
RAIN
SUN

$\rightarrow 3 \times 3 = 9$

$k \times m$

$\omega_B = (\omega_{B1}, \omega_{B2}) = (0, 1)$

IN PRINCIPLE

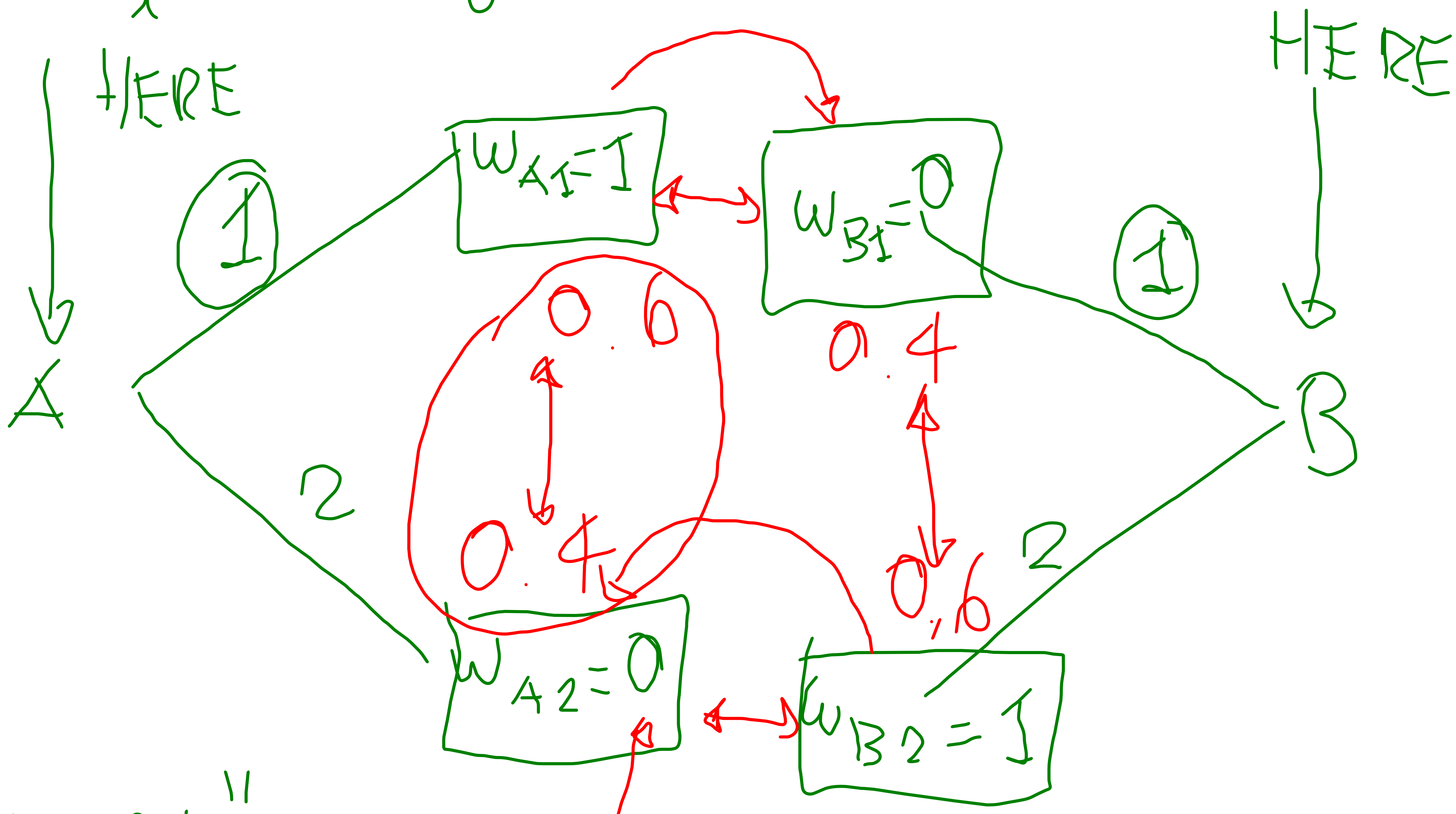
$\pi_A \neq \pi_B$

$B \xrightarrow{\pi_B} 1 \rightarrow$ 0 APPLES

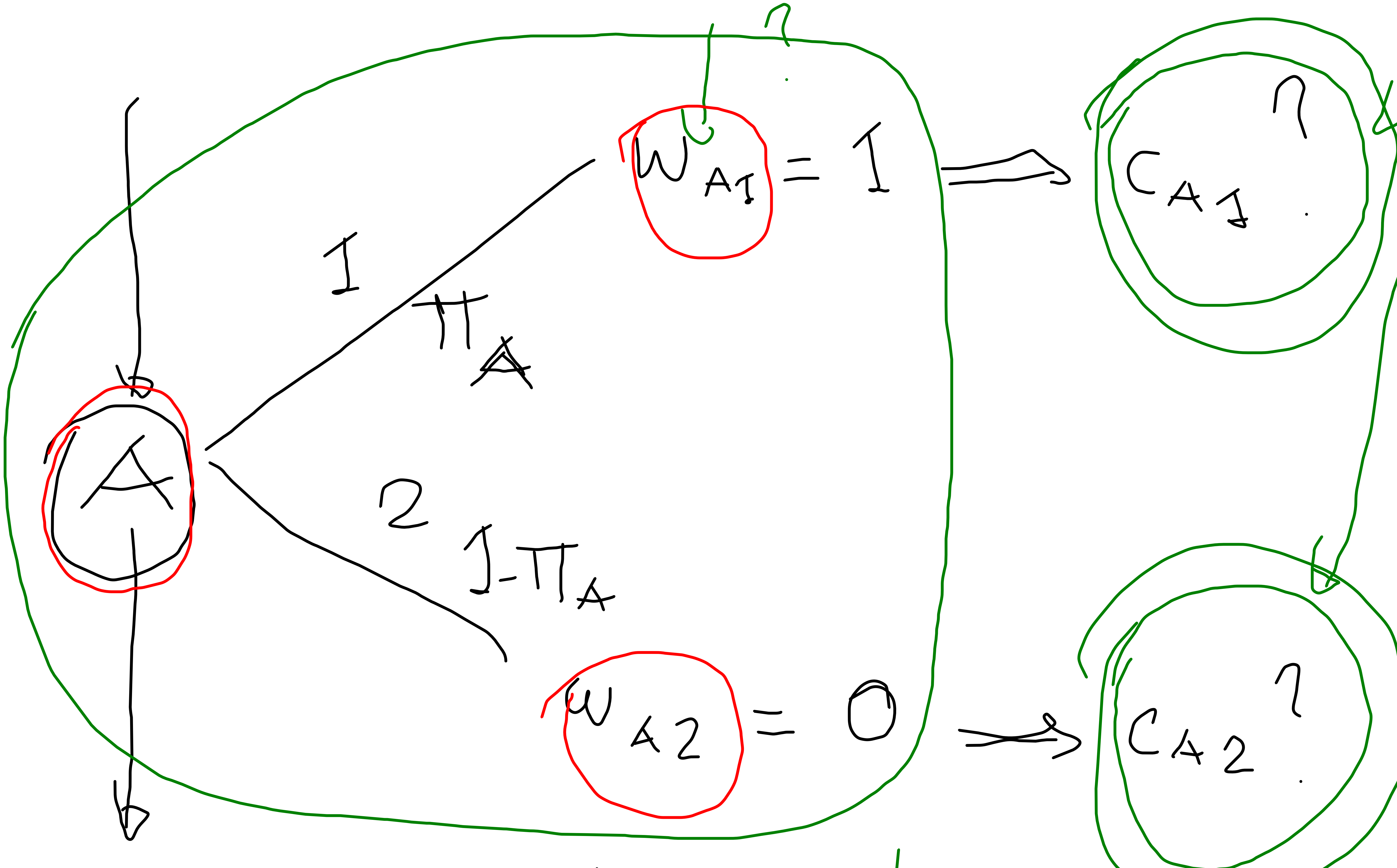
$B \xrightarrow{1-\pi_B} 2 \rightarrow$ 1 APPLE

PREFERENCES

$U_i(c) = \log c \qquad i = A, B$



"HERE" MEANS BEFORE UNCERTAINTY REALISES



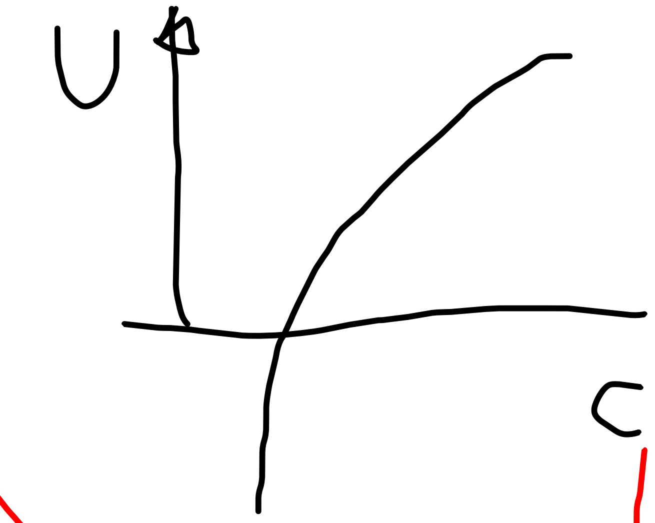
EXPECTED UTILITY

$$\max_{c_{A1},\, c_{A2}} EU_A = \pi_A \log c_{A1} + (1-\pi_A) \log c_{A2}$$

SUBJECT TO $\boxed{p_1 c_{A1} + p_2 c_{A2} \leq p_1 w_{A1} + p_2 w_{A2}}$

$$\max_{c_{A1},\, c_{A2}} EU_A = \pi_A \log c_{A1} + (1-\pi_A) \log c_{A2}$$

SUBJECT TO



$$1 \times c_{A1} + P c_{A2} \le 1 \times 1 + P \times 0 \implies c_{A1} + P c_{A2} \le 1$$

$$P_1 c_{A1} + P_2 c_{A2} \le P_1 \omega_{A1} + P_2 \omega_{A2}$$

WHERE P_1 AND P_2 ARE PRICE OF APPLE IN THE TWO STATES

$P_1 = 1$ $\quad$ $P_2 = P \implies$ JUST ONE PRICE

NUMERAIRE

"EXCHANGE RATE" BETWEEN APPLES IN THE TWO STATES

"APPLES IN STATE 1"

$$\max_{c_{A1},\, c_{A2}} EU_A = \pi_A \log c_{A1} + (1-\pi_A) \log c_{A2}$$

4/2

SUBJECT TO

$$\boxed{c_{A1} + p\, c_{A2} = I}$$

$$p = \frac{\text{APPLES IN STATE 1}}{\text{APPLES IN STATE 2}}$$

EX $p = 2$

$\cancel{0.01}\ 1 = \omega_{A1}$

$\cancel{0.99}\ 0 = \omega_{A2}$

$p = 10$

FIRST ORDER CONDITIONS OF THE LAGRANGIAN

$$\boxed{\frac{\partial L}{\partial c_{A1}} = \frac{\pi_A}{c_{A1}} - \lambda = 0} \rightarrow \boxed{\frac{\pi_A}{c_{A1}} = \lambda}$$

$$\boxed{\frac{\partial L}{\partial c_{A2}} = \frac{1-\pi_A}{c_{A2}} - \lambda p = 0} \rightarrow \boxed{\frac{1-\pi_A}{c_{A2}}} = \boxed{\lambda p}$$

LAGRANGIAN

$$L = EU_A - \lambda(c_{A1} + p\, c_{A2} - I)$$

DIVIDE LEFT HAND SIDE OF THE FIRST COND BY THE LEFT HAND SIDE OF THE SECOND CONDIT. AND EQUALIZE THIS TO THE RATIO OF THE RIGHT HAND SIDES

$$\boxed{C_{A2} = \frac{1-\pi_A}{\cancel{p}}}$$

$$\frac{\frac{\pi_A}{C_{A1}}}{\frac{(1-\pi_A)}{C_{A2}}} = \frac{\quad}{\cancel{p}\; p} \Rightarrow \frac{\pi_A}{C_{A1}} \frac{C_{A2}}{1-\pi_A} = \frac{1}{p}$$

$$\boxed{\frac{p\pi_A C_{A2}}{1-\pi_A} + pC_{A2} = 1}$$

$1 \quad 1$

$$\boxed{C_{A1} = \pi_A}$$

$$C_{A1} = \frac{\cancel{p}\pi_A}{\cancel{1-\pi_A}} \cancel{(1-\pi_A)} / \cancel{p}$$

$$\boxed{C_{A1} = \frac{p\pi_A C_{A2}}{1-\pi_A}} \Rightarrow C_{A1} + pC_{A2} = 1$$

$\max\limits_{c_{B1}, c_{B2}}$ $\boxed{EU_B = \pi_B \log c_{B1} + (1-\pi_B)\log c_{B2}}$

SUBJECT TO

$$p_1 c_{B1} + p_2 c_{B2} = \overset{1 \times 0}{p_1 w_{B1}} + \overset{p \times 1}{p_2 w_{B2}}$$

$$\Downarrow$$

CONSTRAINT $\boxed{c_{B1} + p\,c_{B2} = p \times 1 = p}$

$$L = EU_B - \lambda(c_{B1} + p\,c_{B2} - p)$$

$$\frac{\partial L}{\partial c_{B1}} = \frac{\pi_B}{c_{B1}} - \lambda = 0 \; ; \; \frac{\partial L}{\partial c_{B2}} = \frac{1-\pi_B}{c_{B2}} - \lambda p = 0$$

$$\Longrightarrow \frac{\pi_B / c_{B1}}{1-\pi_B / c_{B2}} = \frac{1}{p} \Longrightarrow \boxed{c_{B1} = \frac{p\pi_B}{1-\pi_B} c_{B2}}$$

$$\frac{p\pi_B}{1-\pi_B} c_{B2} + p\,c_{B2} = \overset{1}{\cancel{p}}$$

$$\boxed{c_{B2} = \cancel{p}(1-\pi_B)}$$

$$\boxed{c_{B1} = p\pi_B}$$

$C_{A1} = \Pi_A$ $\qquad C_{A2} = \dfrac{1-\Pi_A}{p}$

$$\boxed{\begin{array}{l} w_A = \{1, 0\} \\ w_B = \{0, 1\} \end{array}}$$

$C_{B1} = p\Pi_B$ $\qquad C_{B2} = 1 - \Pi_B$

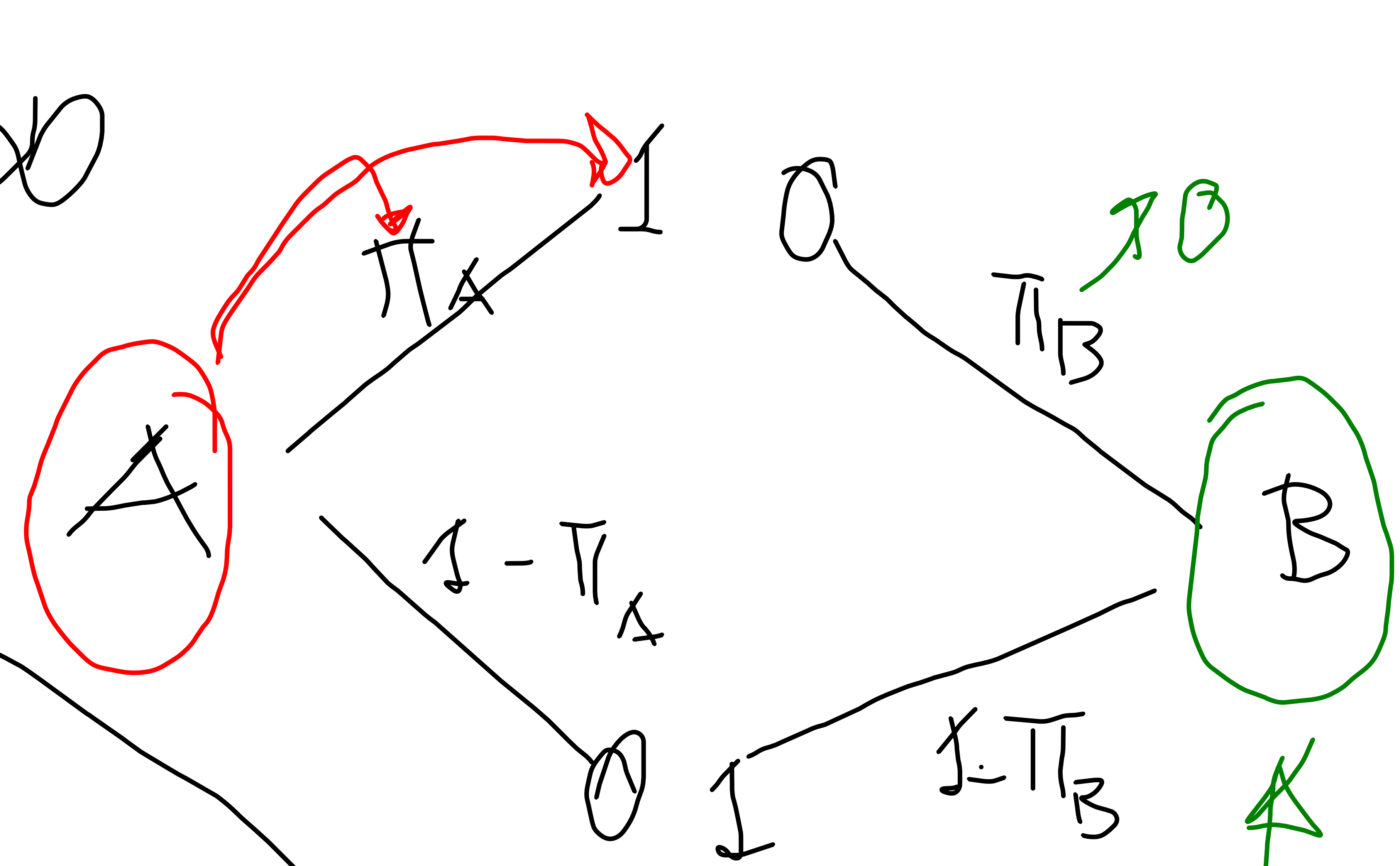
$$\begin{cases} \text{DEMAND} = \text{SUPPLY IN STAGE 1} \\ \quad '' \qquad\qquad '' \qquad\qquad '' \quad 2 \end{cases}$$

$$\begin{cases} C_{A1} + C_{B1} = \Pi_A + p\Pi_B = 1 \\ \dfrac{1-\Pi_A}{p} + \cancel{1} - \Pi_B = \cancel{1} \end{cases} \longrightarrow \boxed{p = \dfrac{1-\Pi_A}{\Pi_B}}$$

$$\lim \boxed{P = \frac{1-\pi_A}{\pi_B}} \to 0 \qquad \to \infty$$

$\pi_B \to 0$

$\pi_A \to 1$



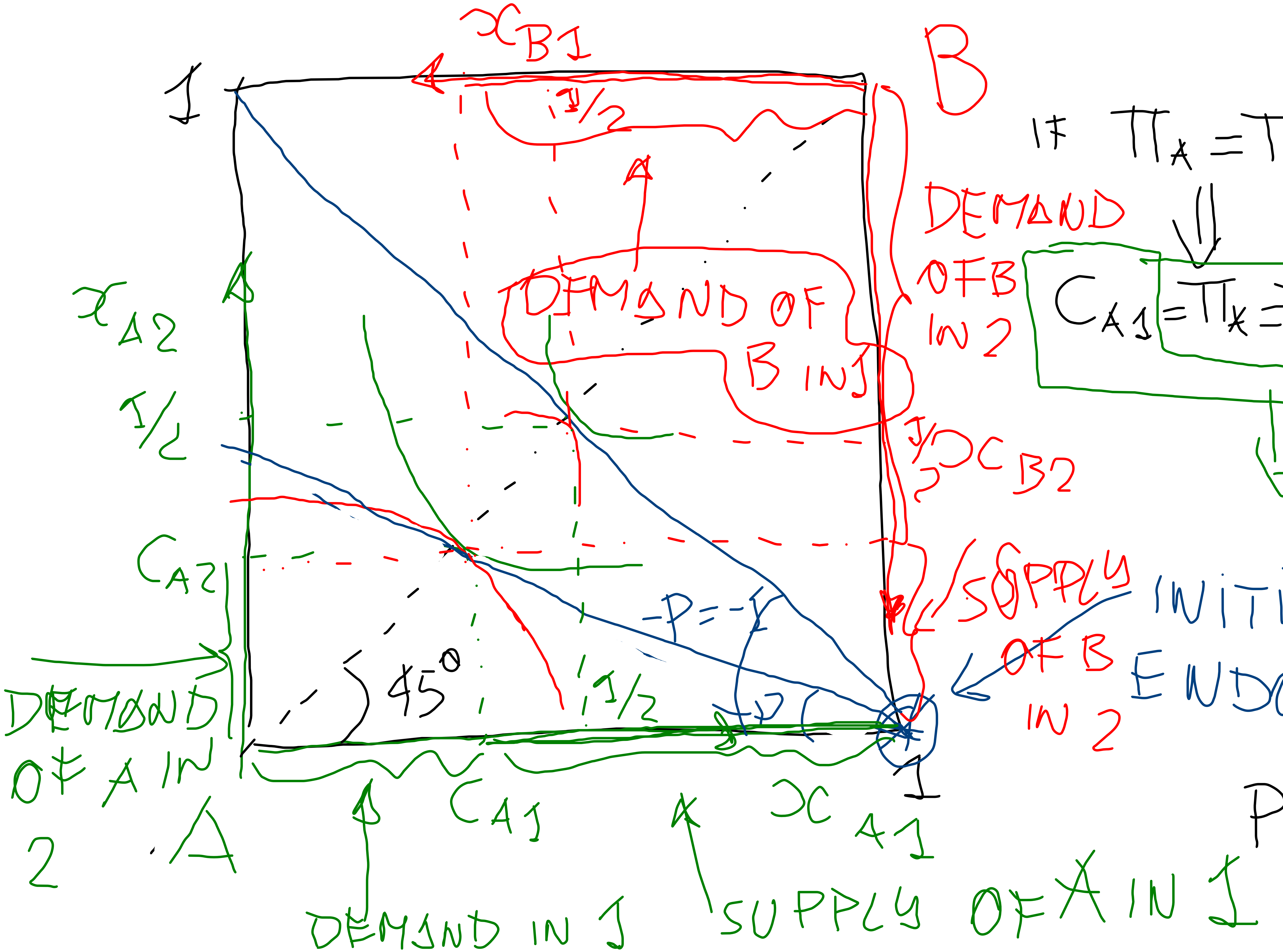
$$P = \frac{\text{APPLES IN } 1}{\text{APPLES IN } 2}$$

$$\boxed{C_{A1} = \pi_A}$$

$$C_{A2} = \frac{1-\pi_A}{P_A} = \frac{1-\pi_A}{(1-\pi_A)/\pi_B} = \pi_B$$

$$\tfrac{1}{2}\quad \pi_A > \pi_B = \tfrac{1}{3} \to \boxed{C_{A1}} > \boxed{C_{A2}} \to \boxed{\pi_A = \pi_B}$$

$$\pi_A < \pi_B$$



If $\pi_A = \pi_B = \boxed{\pi = 1/2}$

$\Downarrow$

$\boxed{C_{A1} = \boxed{\pi_A = \pi_B} = C_{A2}}$

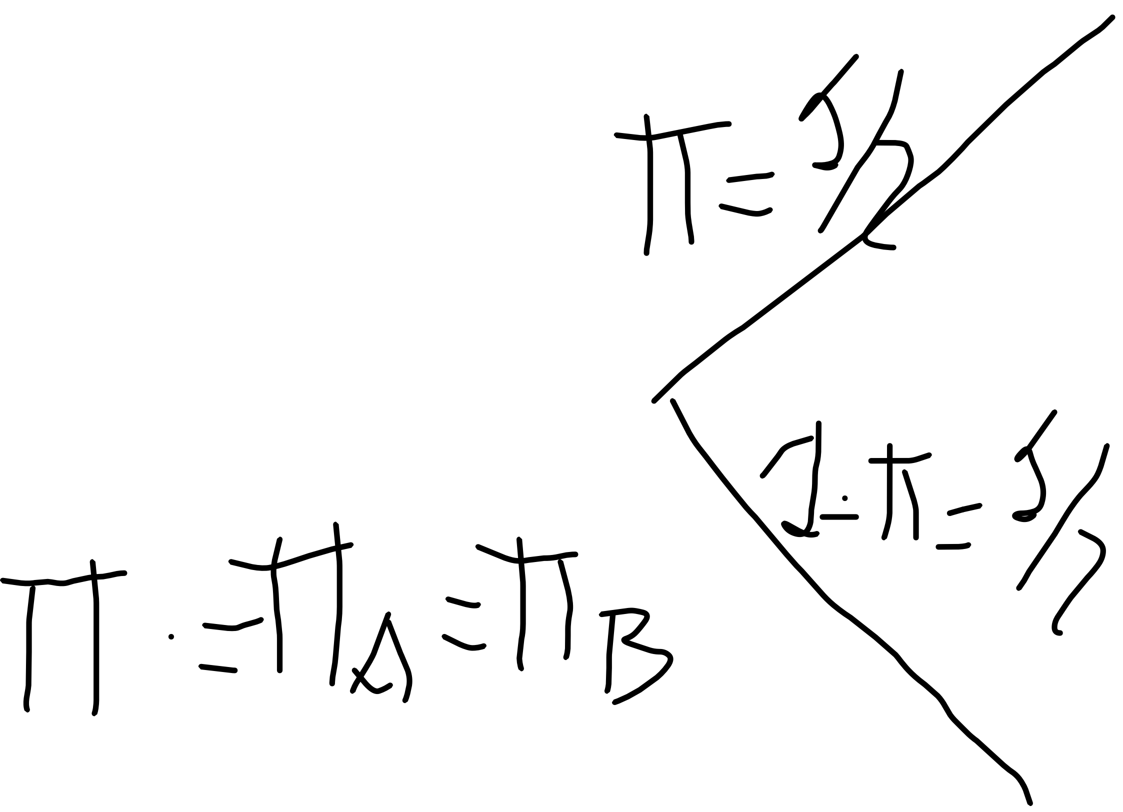
$\Downarrow$

$P = \frac{1-\pi_A}{\pi_B}$

$\Downarrow$

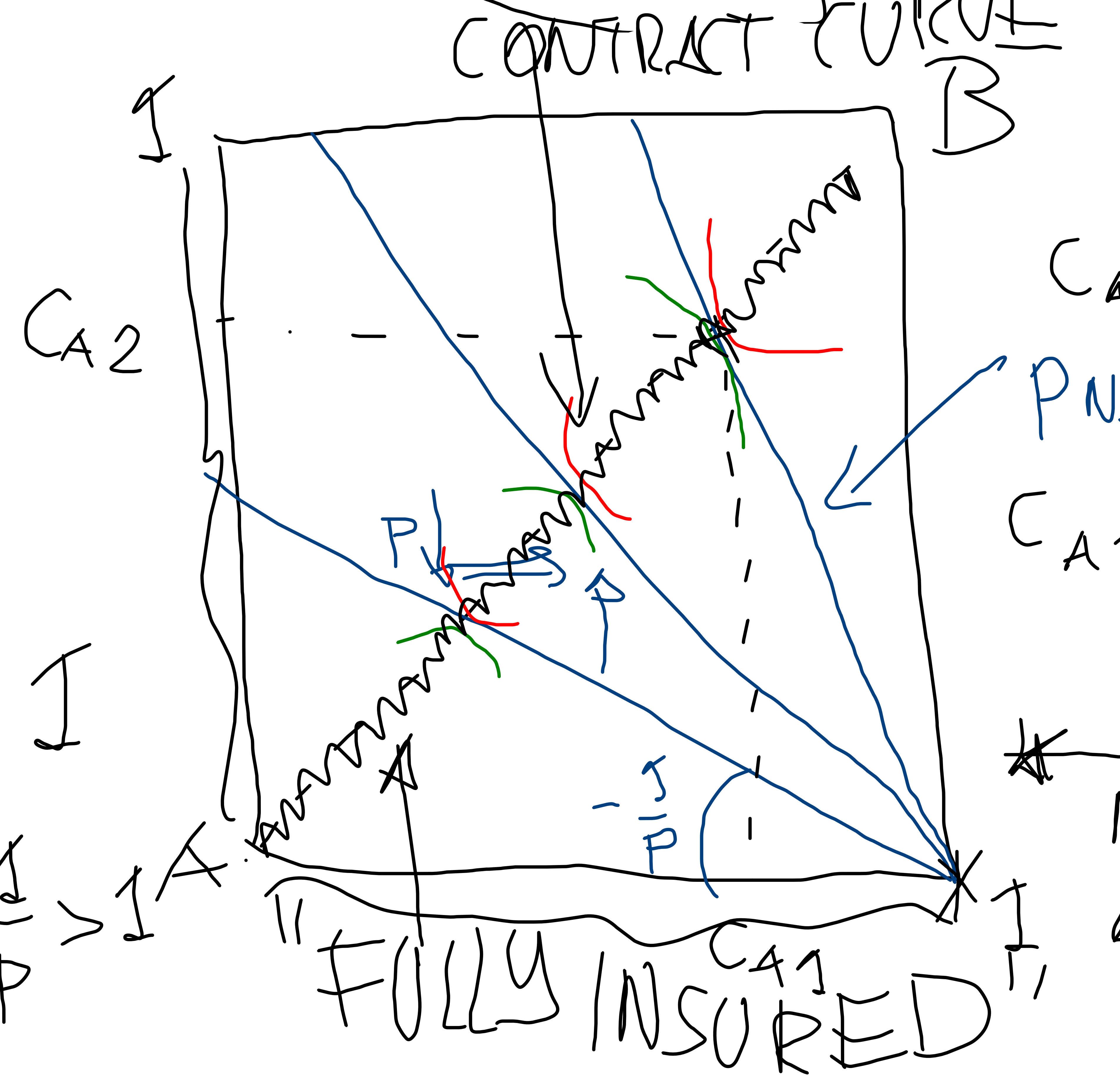
$\frac{1-\pi}{\pi} = 1$

WHY WHEN $\pi = 1/2 \Longrightarrow C_{A1} = C_{A2} = C_{B1} = C_{B2} = 1/2$?



$$P = \frac{1-\pi}{\pi} < 1$$

$$\pi > 1/2 \quad \Downarrow \quad \frac{1}{P} > 1$$



P NEAR 0

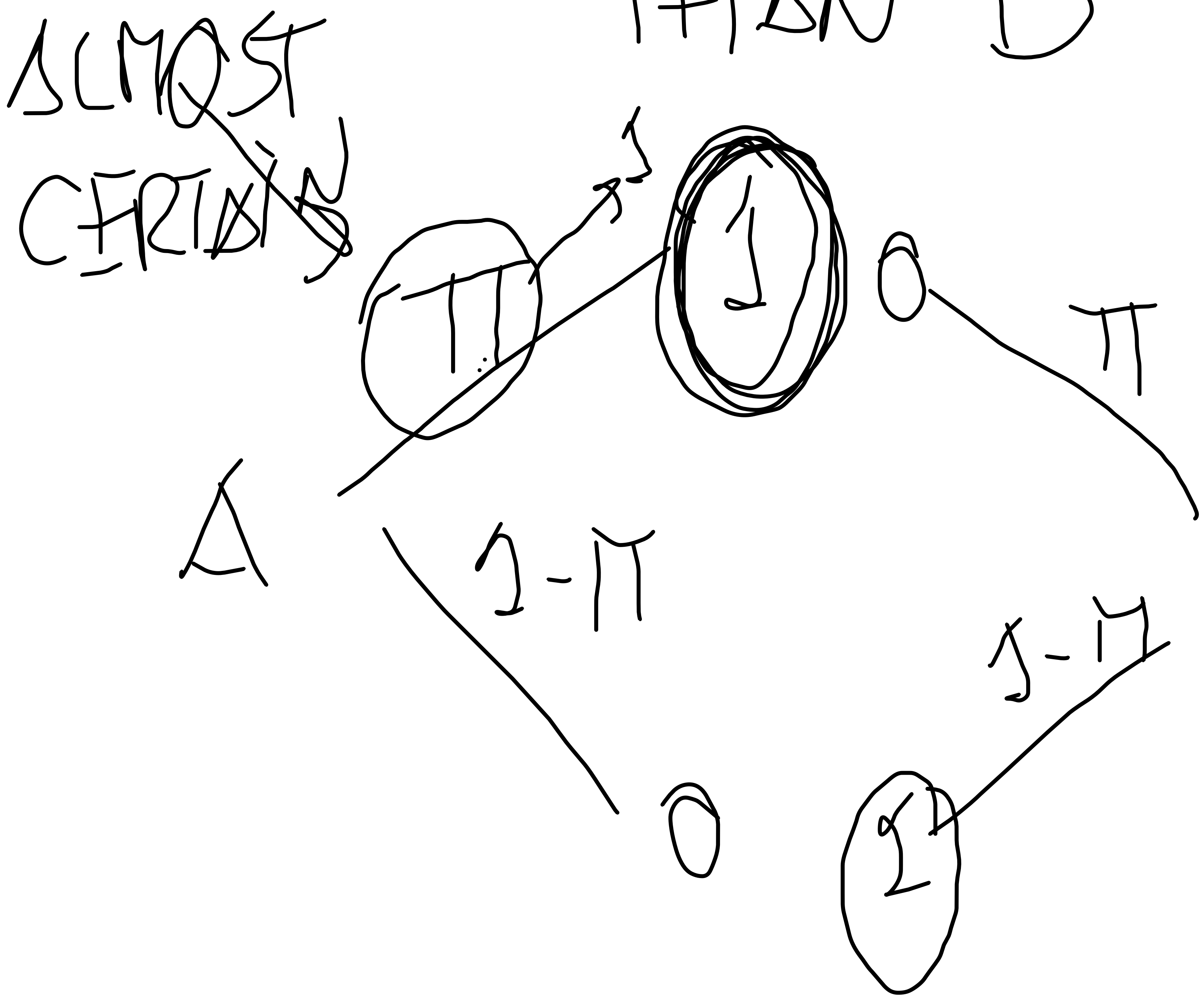
$C_{A1} + PC_{A2} = 1$

$$C_{A2} = \frac{1}{P} - \frac{C_{A1}}{P}$$

NO AGGREGATE UNCERTAINTY

$$\boxed{P = \frac{1-\pi}{\pi}}$$

WHY WHEN $\boxed{P \to 0}$ AGENT A WILL CONSOME MUCH MORE THAN B IN EQUILIBRIUM?



BECAUSE $P \to 0$

IF $\pi \to 1$

$P \to \infty$

$\Rightarrow \pi \Rightarrow 0$

$C_{A1} = \pi_A$ $\qquad$ $C_{A2} = \pi_B$

IF $\pi_A \neq \pi_B \Longrightarrow C_{A1} \neq C_{A2} \Longrightarrow$

$\boxed{\pi_A > \pi_B}$

$\boxed{C_{A1} > C_{A2}}$

$\boxed{P = \frac{1 - \pi_A}{\pi_B}}$

NO FULL INSURANCE

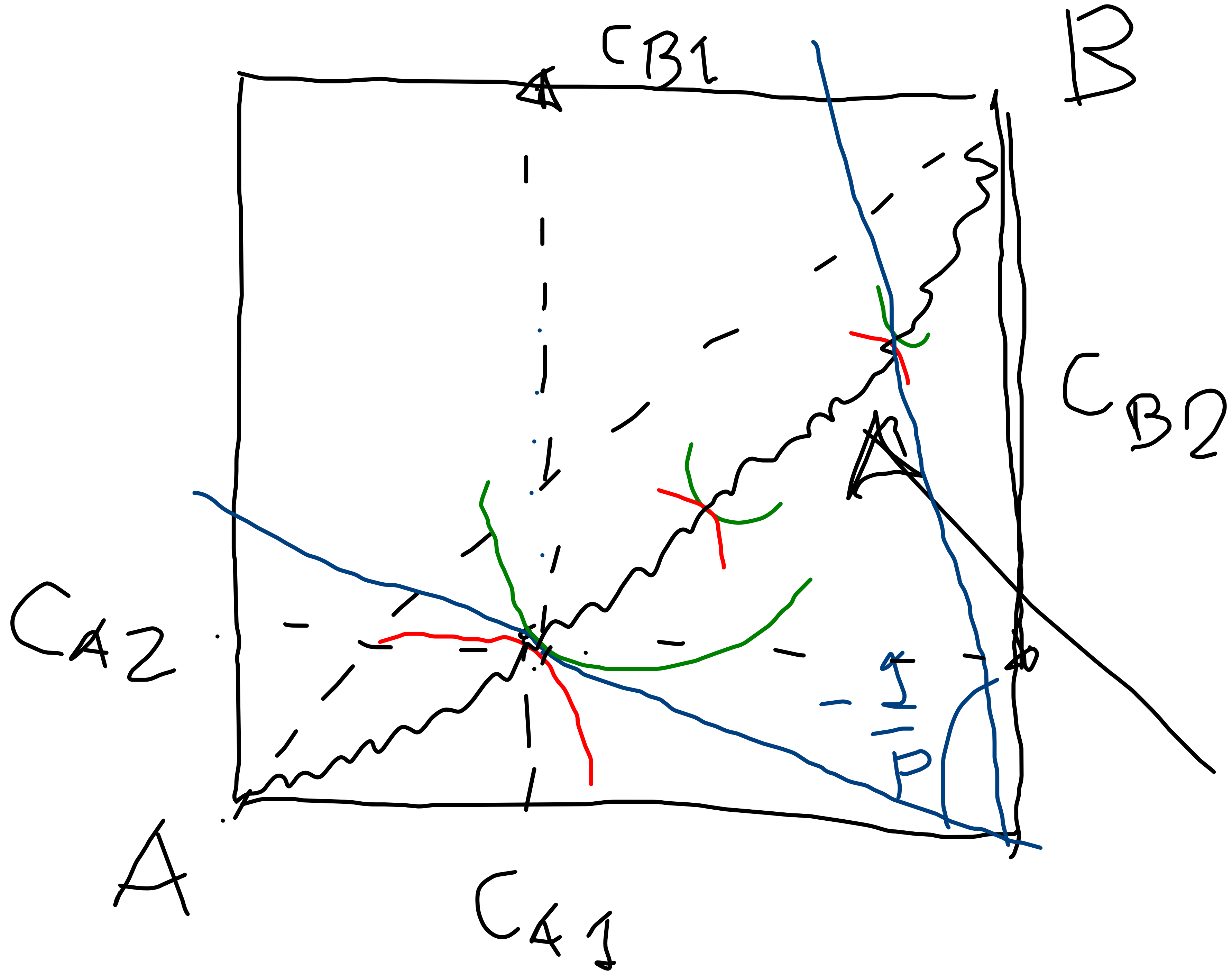
$P \to 0$

NAMELY

$\pi_A \to 1$

OR

$\pi_B \to 0$



CONTRACT CURVE

$\boxed{C_{B2} > C_{B}}$

HOMEWORK

FIND THE EQUILIBRIUM ALLOCATIONS + PRICE

1) $U(c) = \log c \longrightarrow$ CONCAVITY

1 COMMODITY

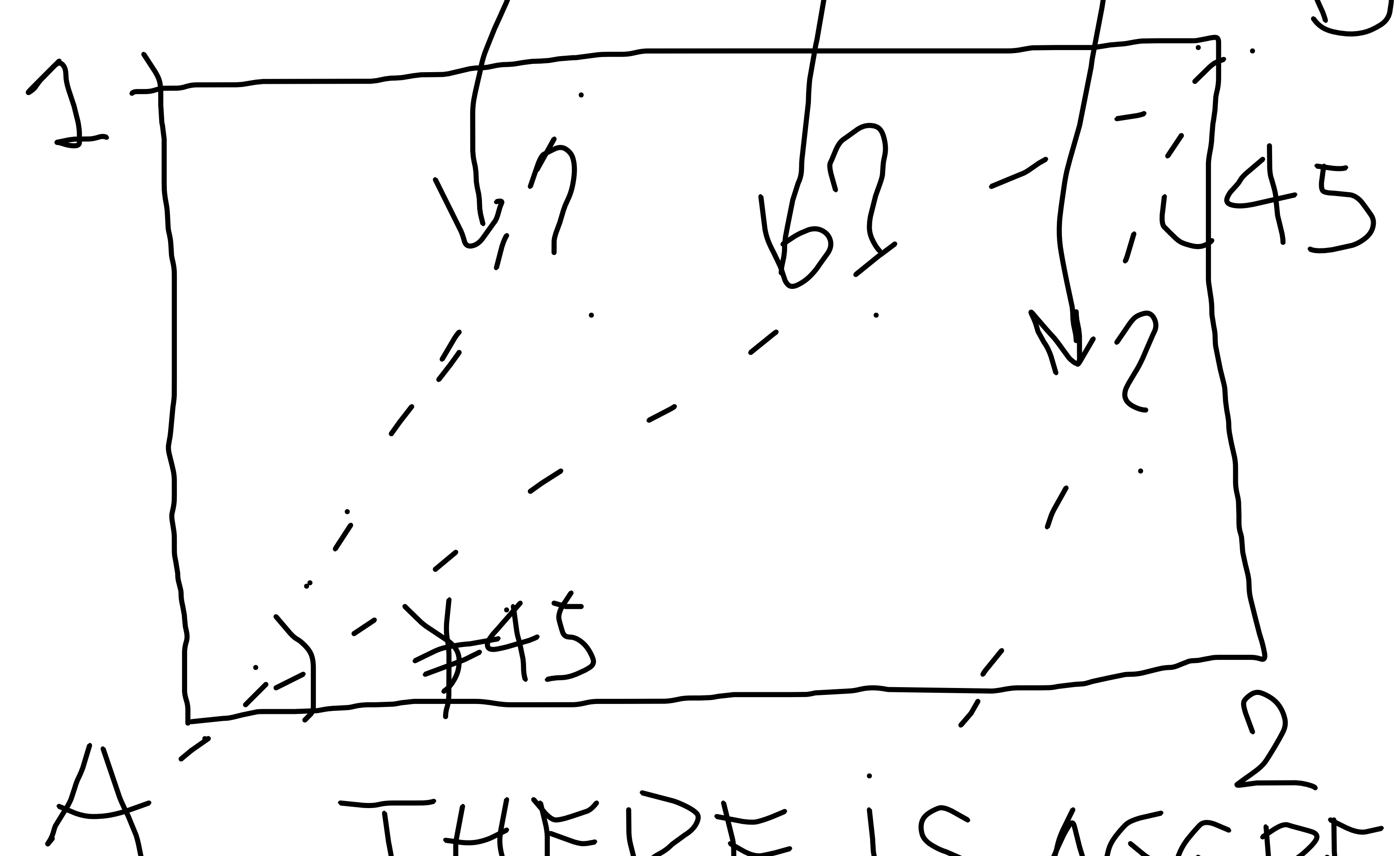
2 AGENTS _ 2 STATES

$\pi_A = \pi = \pi_B$

$w_A = (2, 0)$

$w_B = (0, 1)$

EQ ALLOCA



THERE IS AGGREGATE UNCERTAIN

2) $U(c) = c_A$ LINEAR BE CAREFUL BECAUSE TYPICALLY 1ST ORDER CONDITION DOES NOT GIVE THE ANSWER

2 INDIV.

2 STATES

$\omega_A = (1, 0)$

$\omega_B = (0, 1)$

$\pi_A = \pi = \pi_B$

FIND EQ. ALL + PRICE

$$EU_A = \pi c_{A1} + (1-\pi) c_{A2}$$

SUBJECT TO

$$c_{A1} + p c_{A2} = 1$$

$$EU_B = \pi c_{B1} + (1-\pi) c_{B2}$$

SUBJECT TO

$$c_{B1} + p c_{B2} = p$$