A.S.

BUYER - SELLER

SYMMETRIC COMPAREIT WITH ASYMM INFO

PREFERENCE PAR

BUYER $\boxed{U(q, T, \theta) = \theta v(q) - T}$

$v' > 0, v'' < 0$ WHERE q = QUANTITY

T = TOTAL SUM PAID FOR QUANTITY q

$v(0) = 0$ θ COULD BE HIDDEN BY THE BUYER TO THE SELLER

WE ASSUME <u>TWO TYPES</u> OF BUYERS IN THE POPULATION

$$\theta = \begin{cases} \theta_H = \text{HIGH} \\ \theta_L = \text{LOW} \end{cases}$$

$$P(\theta_H) = 1 - P(\theta_L) = 1 - \beta$$

$$\theta_H > \theta_L$$

$$\longrightarrow 0 \leq P(\theta_L) = \beta \leq 1$$

$$\Theta = \{\theta_L, \theta_H\}$$

SELLER

REVENUES

COSTS

$$\boxed{\Pi(q,T) = T - cq}$$

c = UNIT (MARGINAL COST) > 0

AMOUNT RECEIVED WHEN SELLING q UNITS OF THE GOOD

QUESTION

HOW T AND q BE DETERMINED BY THE SELLER IN AN OPTIMAL WAY?

NOTE (T, q) DEFINE "A CONTRACT" WHICH IS DESIGNED (PROPOSED) BY THE SELLER

ANSWER

THE OPTIMAL CONTRACT DEPENDS ON THE INFORMATION DISTRIBUTION ON θ

SYMMETRIC INFORMATION

$$\max_{T_i, q_i} N\beta[T_L - cq_L] + N(1-\beta)[T_H - cq_H]$$

SUCH THAT $\theta_i v(q_i) - T_i \geq U_i = 0$

$i \in \{L, H\}$

N PEOPLE

$N\beta$

PARTICIPATION CONSTRAINT

$$\boxed{\max_{T_i, q_i} \; \beta\left[T_L - cq_L\right] + (1-\beta)\left[T_H - cq_H\right]}$$

SUCH THAT

$$\boxed{\begin{aligned} \theta_L V(q_L) - T_L \geq 0 \quad & PC_L \\ \theta_H V(q_H) - T_H \geq 0 \quad & PC_H \end{aligned}}$$

PARTICIPATION CONSTRAINTS

4 VARIABLES (T_L, q_L) (T_H, q_H)

<u>SOLUTION</u>

$PC_L \Longrightarrow \theta_L V(q_L) - T_L = 0 \longrightarrow (T_L, q_L)$

$(*) \Longrightarrow \boxed{\theta_L V(q_L) = T_L}$

SATISFY EQUALITY

SUPPOSE IT DOES NOT

$\boxed{\theta_L V(q_L) - T_L > 0} \longrightarrow$ THE CONTRACT IS NOT OPTIMAL BECAUSE THE SELLER $T_L \uparrow$

$(**) \Longrightarrow \boxed{\theta_H V(q_H) = T_H}$ PC_H

$$\max_{q_L, q_H} \Pi = \beta\Big[\overbrace{\underset{\uparrow\, PC_L}{\theta_L} V(q_L)}^{T_L} - c q_L\Big] + (1-\beta)\Big[\overbrace{\underset{\uparrow\, PC_H}{\theta_H} V(q_H)}^{T_H} - c q_H\Big]$$

$$\frac{\partial \Pi}{\partial q_L} = \beta\left[\theta_L V'(q_L) - c\right] = 0 \Rightarrow \theta_L V'(q_L) = c$$

FOC

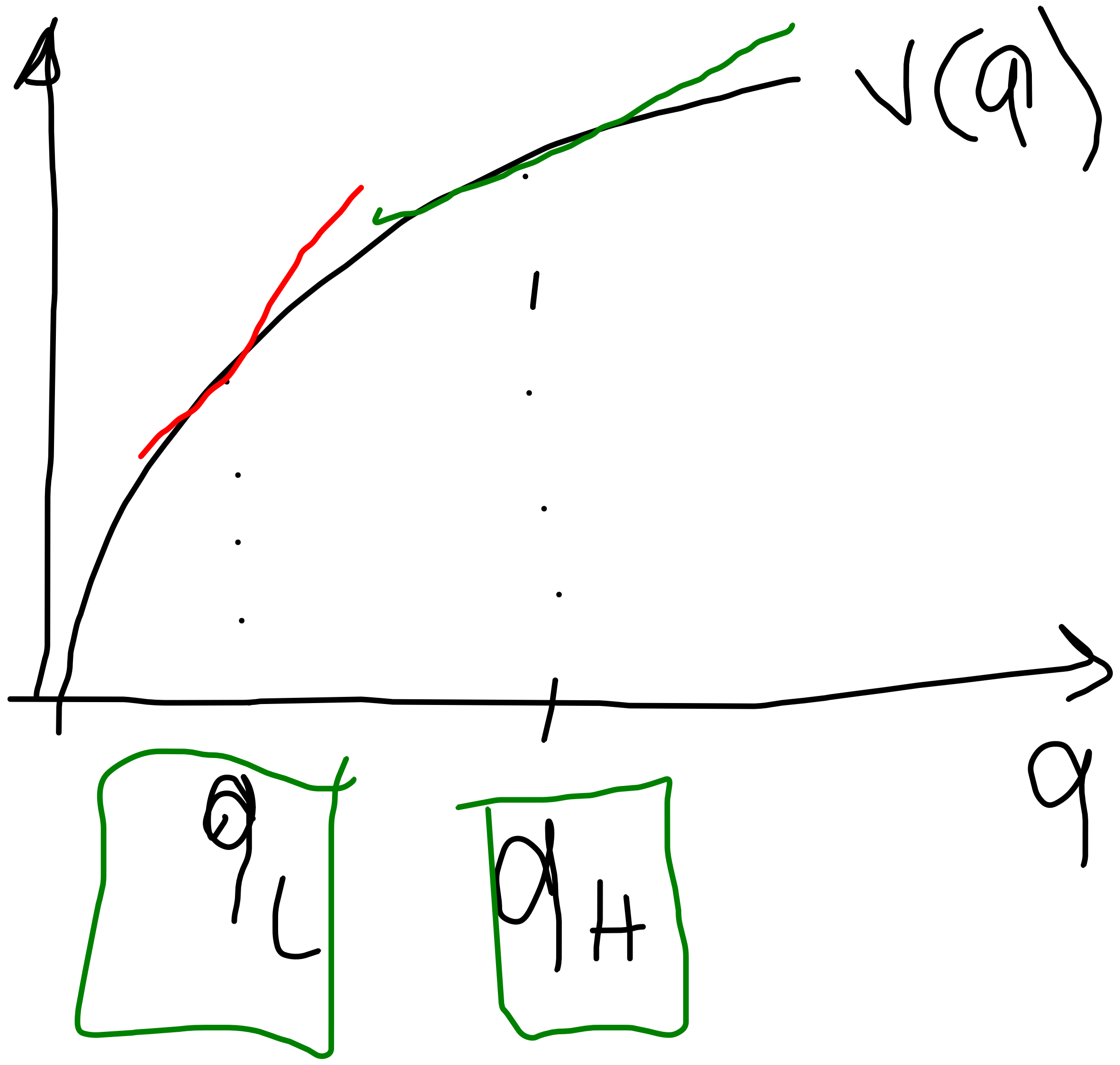
$$\frac{\partial \Pi}{\partial q_H} = (1-\beta)\left[\theta_H V'(q_H) - c\right] = 0 \Rightarrow \theta_H V'(q_H) = c$$

FOC $\Longrightarrow \boxed{\theta_L V'(q_L) = \theta_H V'(q_H)}$

$$\boxed{\theta_L v'(q_L) = \theta_H v'(q_H)} \Longrightarrow ?$$

Since $\theta_H > \theta_L \Longrightarrow v'(q_L) > v'(q_H) \Longrightarrow q_L < q_H$

BECAUSE OF THE CONCAVITY OF V



$\boxed{T_L, T_H ?}$

PC$_L$ $\theta_L v(q_L) = T_L$

PC$_H$ $\theta_H v(q_H) = T_H$

EXAMPLE

$$v(q) = \sqrt{q} \qquad U(T, q, \theta) = \theta\sqrt{q} - T$$

$$\theta \in \{\theta_L, \theta_H\}$$

$$FOC_L \Rightarrow \theta_L v'(q_L) = c \Longrightarrow \theta_L \frac{1}{2\sqrt{q_L}} = c$$

$$\Longrightarrow \boxed{q_L^* = \left(\frac{\theta_L}{2c}\right)^2}$$

SO IT IS COMPUTED BECAUSE θ_L AND c ARE KNOWN TO THE SELL[ER]

$$\Longrightarrow T_L^* ? \quad PC_L \Rightarrow \theta_L \sqrt{q_L^*} = T_L^*$$

$$\Longrightarrow \boxed{T_L^* = \theta_L \sqrt{\left(\frac{\theta_L}{2c}\right)^2} = \frac{\theta_L^2}{2c}}$$

$$q_H^* = \left(\frac{\theta_H}{2c}\right)^2$$

$$T_H^* = \theta_L^2 / 2c$$

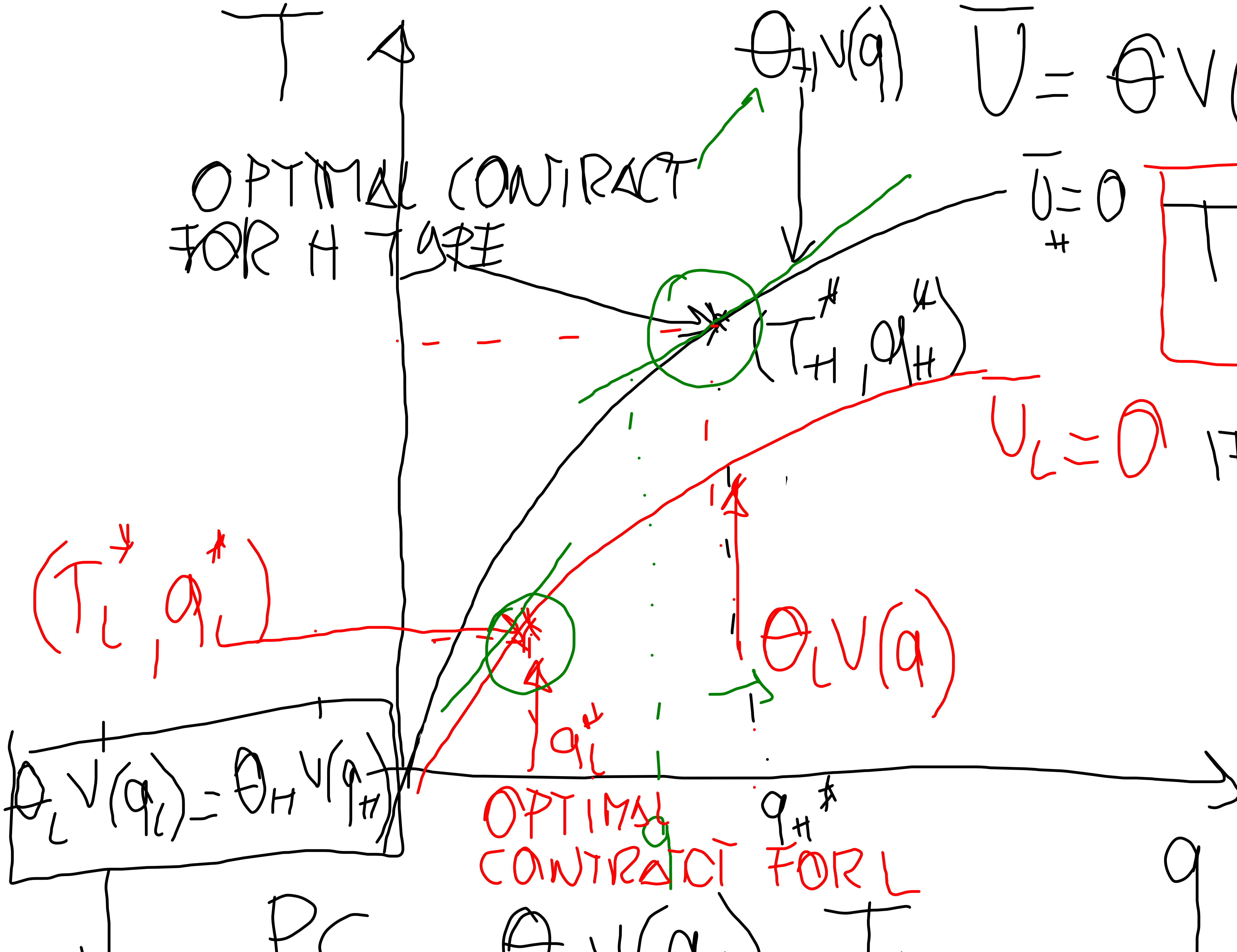
$$q_L^* = \left(\frac{\theta_L}{2c}\right)^2 \quad T_L^* = \frac{\theta_L^2}{2c} \qquad q_H^* = \left(\frac{\theta_H}{2c}\right)^2 \quad T_H^* = \frac{\theta_H^2}{2c}$$

$$U_L(\theta_L, T_L, q_L) = \theta_L\sqrt{q_L^*} - T_L^* = 0 \Rightarrow \theta_L\sqrt{\left(\frac{\theta_L}{2c}\right)^2} - \frac{\theta_L^2}{2c} = \frac{\theta_L^2}{2c} - \frac{\theta_L^2}{2c} = 0$$

$$U_H(\theta_H, T_H^*, q_H^*) = 0$$

$$\Pi = \beta\left[T_L^* - cq_L^*\right] + (1-\beta)\left[T_H^* - cq_H^*\right] =$$

$$= \beta\left[\frac{\theta_L^2}{2c} - c\left(\frac{\theta_L}{2c}\right)^2\right] + (1-\beta)\left[\frac{\theta_H}{2c} - c\left(\frac{\theta_H}{2c}\right)^2\right]$$



$\bar{U} = \theta V(q) - T$

$\boxed{T = \theta V(q) - \bar{U}}$

IF $\bar{U} = 0$

$\Rightarrow \boxed{T = \theta V(q)}$

AND SINCE

$V(0) = 0$

$\Rightarrow T = \theta V(q)$

GOES THROUGH THE ORIGIN

$PC_L \quad \theta_L V(q_L) = T_L$

$P_H \quad \theta_H V(q_H) = T_H$

NEXT STEP: WE ARE GOING TO COMPARE THE OPTIMAL $(T_L^*, q_L^*), (T_H^*, q_H^*)$ CONTRACT WITH SYMMETRIC INFO

WITH THE OPTIMAL CONTRACT $(T_L', q_L'), (T_H', q_H')$ UNDER ASYMMETRI