SYMMETRIC INFO (FIRST BEST CONTRACT)

BUYER - SELLER MODEL

$$U = \boxed{\theta} v(q) - T$$

PIECE OF INFO THAT MAY BE HIDDEN

ASYMMETRIC INFO (SECOND BEST)

$$\max_{T_i, q_i} \; \beta\left[\theta_i v(q_i) - T_i\right] + (1-\beta)\left[\theta_H v(q_H) - T_H\right]$$

SUCH THAT

$$\begin{cases} \theta_L v(q_L) - T_L \geq 0 & \text{PCL} \\ \theta_H v(q_H) - T_H \geq 0 & \text{PCH} \end{cases}$$

$$\begin{cases} \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L & \text{ICH} \\ \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H & \text{ICL} \end{cases}$$

L L

H H H

INCENTIVE COMPATILITY CONSTRAINTS

SOLUTION

NOTE PROPOSING ~~T~~ THE FBC IS NOT OPTIMAL NOW

STEP 1

PCH WILL NOT BIND AT THE OPTIMUM AND COULD INITIALLY BE ELIMINATED

$\theta_H > \theta_L$ PCL

ICH $\boxed{\theta_H V(q_H) - T_H} \geq \theta_H V(q_L) - T_L > \boxed{\theta_L V(q_L) - T_L} \geq 0$

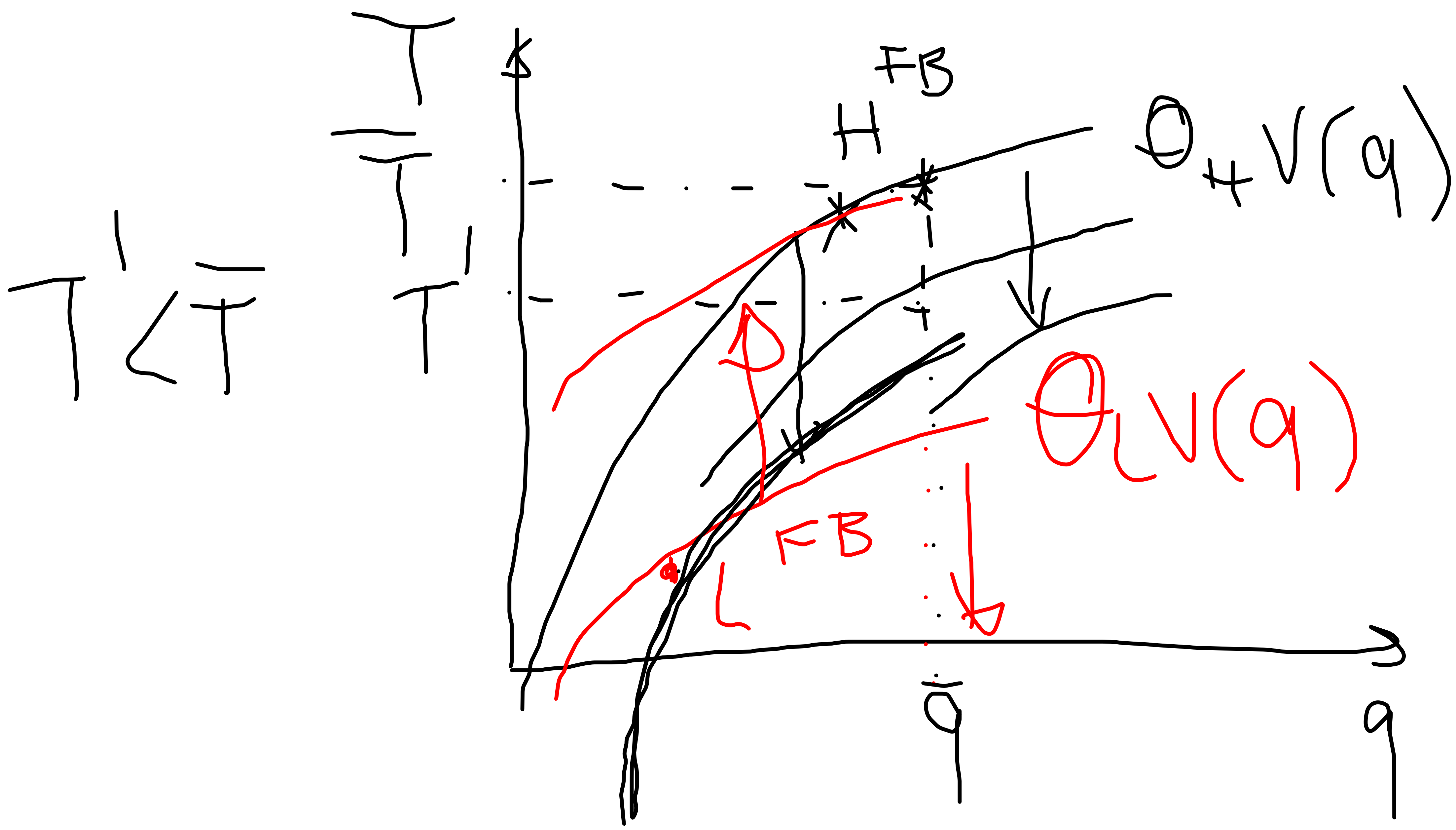
THE SAME

ONLY θ CHANGES

$\Rightarrow \boxed{\theta_H V(q_H) - T_H > 0} \,!$

STEP 2

REMOVE INITIALLY ICL. WHY? BECAUSE TYPICALLY IS THE H TYPE THAT COULD PRETEND TO BE A L TYPE RATHER THAN VICEVERSA



STEP 3

$$\max_{T_i, q_i} \beta\left[T_L - cq_L\right] + (1-\beta)\left[T_H - cq_H\right]$$

PCL $\quad \theta_L V(q_L) - T_L \geq 0$

ICH $\quad \theta_H V(q_H) - T_H \geq \theta_H V(q_L) - T_L$

NOTICE THAT AT THE OPTIMUM (SBC) PCL AND ICH ARE SATISFIED WITH = (THEY BIND)

SUPPOSE PCL IS SATISFIED WITH $\theta_L V(q_L) - T_L > 0 \Longrightarrow$ NOT OPTIMAL

BECAUSE THE SELLER CAN ALWAYS INCREASE BY A SMALL AMOUNT T_L AND STILL SATISFY PCL

$$\theta_L V(q_L) - T_L > 0$$

$T_L' > T_L$ WHERE T_L' IS SLIGHTLY LARGER THAN T_L

$\Rightarrow$ PCL $=$

$$\theta_L V(q_L) = T_L \text{ AT THE OPTIMUM}$$

BY A SIMILAR ARGUMENT IT CAN NOT BE

ICH $\quad \theta_H V(q_H) - T_H \geq \theta_H V(q_L) - T_L$

$=$

STEP 4

$$\max_{T_i, q_i} \ \beta[\boxed{T_L} - cq_L] + (1-\beta)[T_H - cq_H]$$

S T

U H

C J

H T

$$\theta_L V(q_L) - T_L = 0 \quad \text{PCL} \rightarrow \boxed{T_L = \theta_L V(q_L)}$$

$$\theta_H V(q_H) - T_H = \theta_H V(q_L) - T_L \quad \text{ICH}$$

$$\Downarrow$$

$$\boxed{T_H = \theta_H V(q_H) - \theta_H V(q_L) + T_L}$$

$$\Longrightarrow \boxed{T_H = \theta_H V(q_H) - \theta_H V(q_L) + \theta_L V(q_L)}$$

ONCE REPLACED T_L AND T_H ACCORDING TO ~~THE EXPRE~~ CONSTRAINTS SSIONS IN THE PREVIOUS SLIDE THE PROBLEM BECOMES

$$\boxed{\max_{q_L, q_H} \beta\left[\theta_L v(q_L) - c q_L\right] + (1-\beta)\left[\theta_H v(q_H) - c q_H - (\theta_H - \theta_L) v(q_L)\right]}$$

$\Rightarrow$ 2 VARIABLES NO CONSTRAINTS ~~PROB~~ PROBLEM ASSUMING INTERNAL SOLUTIONS EXIST FOR A MAX $(q_L, q_H) > 0$ WE CAN SOLVE THE PROBLEM TAKING FOC $\Rightarrow (q_L^*, q_H^*)$

$$\frac{\partial \text{ EXPECTED PROFIT}}{\partial q_H} = (1-\beta)\left[\theta_H v'(q_H) - c\right] = 0 \qquad (1)$$

$$\frac{\partial \text{ EXPECTED PROFIT}}{\partial q_L} = \beta\left[\theta_L v'(q_L) - c\right] + (1-\beta)\left[\theta_L - \theta_H\right] v'(q_L) = 0$$

$$- (1-\beta)\left[\theta_H - \theta_L\right] v'(q_L) = 0 \qquad (2)$$

$$(1) \Longrightarrow \theta_H v'(q_H^{SB}) = c \Longrightarrow \boxed{q_H^{SB} = q_H^{FB}} \;!$$

$$(2) \quad \theta_L v'(q_L^{SB}) = \frac{c}{1 - \left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right)} > c$$

$$0 < 1 - \left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right) < 1$$

① $\theta_H v'(q_H^{SB}) = c \Longrightarrow \boxed{q_H^{SB} = q_H^{FB}}$

② $\theta_L v'(q_L^{SB}) = \dfrac{c}{1 - \left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right)} > c \Longrightarrow \boxed{q_L^{SB} < q_L^{FB}}$

$0 < 1 - \left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right) < 1$

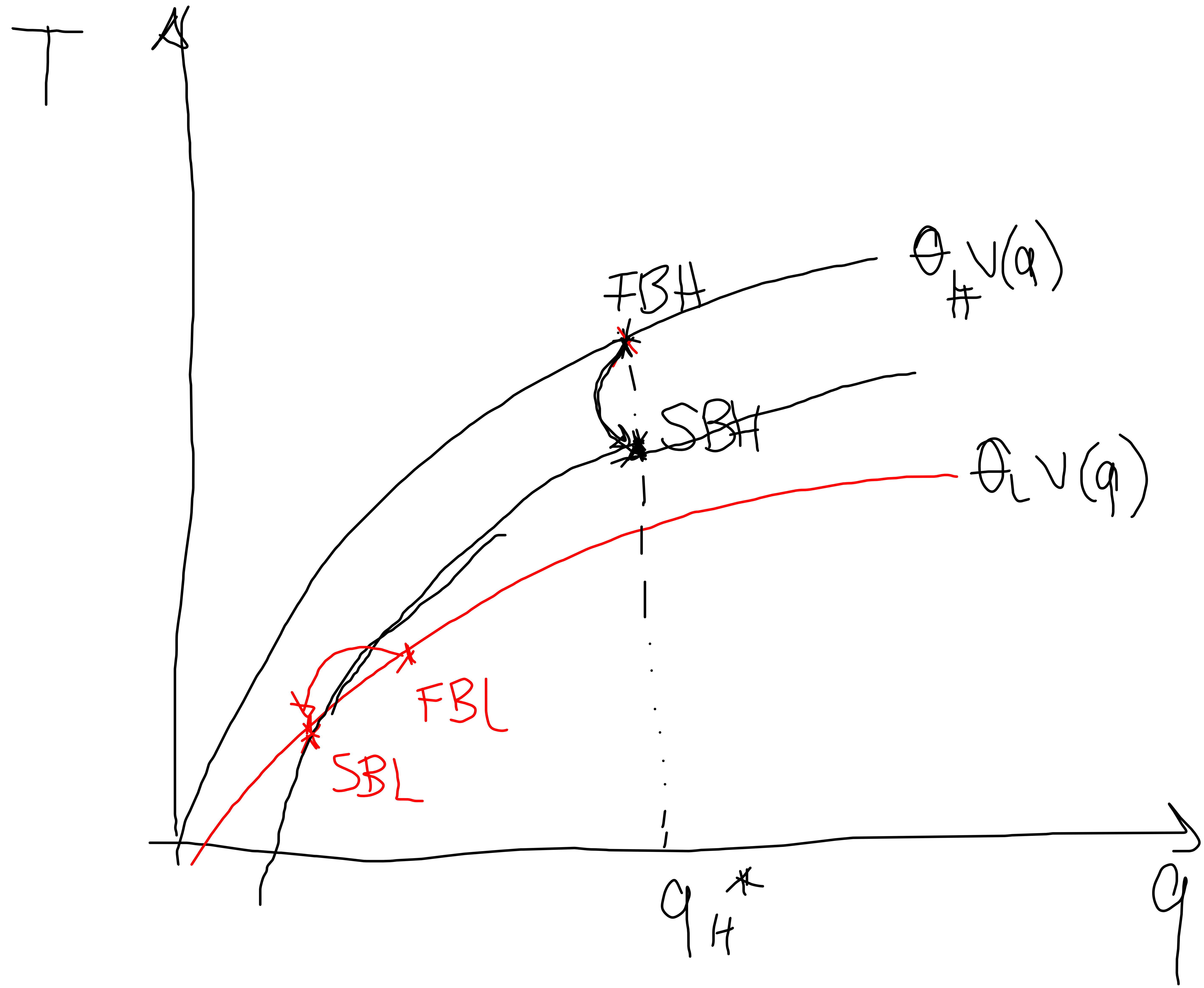
DISTORTION IN CONSUMPTION OF L TYPE TO DISCOURAGE H TYPES TO CHOOSE q_L^{SB}

OBSERVATIONS

(a) THE QUANTITY $(\theta_H - \theta_L) > 0$ IS CALLED THE "INFORMATIONAL RENT" OF THE H TYPE. IF $\theta_H - \theta_L$ GETS LARGE $\Longrightarrow \dfrac{c}{1 - \boxed{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right)}} \longrightarrow \infty$

$\boxed{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right)} \longrightarrow 1$

$\Longrightarrow q_L^{SB} \longrightarrow 0$



T
FBH
SBH
$\theta_H V(q)$
$\theta_L V(q)$
FBL
SBL
q_H^*
q