UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2021/22
Quantitative Methods for Economic Applications - Mathematics for Economic Applications
Task 7/2/2022

I M 1) Find all the sixth order roots of the real number -64.

I M 2) Consider the matrix: $\mathbb{A} = \begin{bmatrix} a & 1 & 2 \\ 1 & b & 2 \\ 1 & 2 & c \end{bmatrix}$. Knowing that $(1,1,1)$ is an eigenvector of $\mathbb{A}$ associated to the eigenvalue $\lambda = 1$, find the values of the three parameters a, b and c and check if the matrix is a diagonalizable one.

I M 3) Given the linear map $F\colon \mathbb{R}^3 \to \mathbb{R}^3$, with $F(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, kx_1 + mx_3)$; knowing that the dimention of the Kernel of F is 1, find the relation between the parameters k and m, and choose a pair (k, m) that satisfy the relation before found, calculate a basis for the Immage of F and a basis for the Kernel of F.

I M 4) Check if there are values of x and y for wich the matrix $\mathbb{A} = \begin{bmatrix} 2 & x \\ y & 1 \end{bmatrix}$ is similar to the matrix $\mathbb{B} = \begin{bmatrix} 11 & 29 \\ -3 & -8 \end{bmatrix}$, if the matrix of the similarity transformation is $\mathbb{P} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

II M 1) Given the equation $f(x,y,z) = (x + y - z)e^{x - yz} - (x - y + z)e^{y - xz} = 0$ satisfied at the point $(1,1,1)$, verify that with it an implicit function $z = z(x,y)$ can be defined and then calculate, for this implicit function, its gradient.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = xy \\ \text{u.c.: } x^2 + y^2 \leq 4 \end{cases}$.

II M 3) Check if the function $f(x,y) = (|x| + |y|)(x + y)$ is differentiable at $(0,0)$.

II M 4) Given $f(x,y) = (x - y)^2 + (x + y)^2$ and the unit vectors $v = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$ and $w = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$, determine all the points where $\mathcal{D}_v f(x,y) = \mathcal{D}_w f(x,y)$.