

UNIVERSITA' DEGLI STUDI DI SIENA

Facoltà di Economia "R. Goodwin"

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Quantitative Methods for Economic Applications - Mathematics for Economic Applications

Task 10/1/2022

I M 1) Given two complex numbers, z_1 and z_2 ; both numbers have modulus equal to $\sqrt{2}$, the first z_1 has argument $\frac{2}{3}\pi$, while z_2 has argument $\frac{1}{3}\pi$. Write the two numbers in algebraic form and find the cubic roots of the complex number $z = \frac{z_1}{z_2}$.

I M 2) Consider the matrix: $\mathbb{A} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & k & 1 \end{bmatrix}$. Knowing that $\lambda = 1$ is an eigenvalue of \mathbb{A} ,

find the value of k , and check if the matrix is diagonalizable or not.

I M 3) Given a linear map $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3$, with

$F(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_2 + x_3, kx_3 + kx_4)$, find the value of k that maximizes the dimension of the Kernel of F , and with such value of k calculate a basis for the Image of F .

I M 4) Given the linear homogeneous system
$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 0 \\ mx_1 + x_2 + x_3 + x_4 + mx_5 = 0 \\ x_1 + kx_2 + kx_3 + kx_4 + x_5 = 0 \end{cases}$$
, find, on

varying the parameters k and m , the dimension of the linear space of its solutions.

II M 1) Given the equation $f(x, y) = e^{x^2-y^2} - (1 + x^4 - y^4) = 0$ satisfied at the point $(1, 1)$, verify that with it an implicit function $y = y(x)$ can be defined and then calculate, for this implicit function, its first and second order derivatives.

II M 2) Solve the problem
$$\begin{cases} \text{Max/min } f(x, y) = 3x - 2y \\ \text{u.c.: } x^2 \leq y \leq x \end{cases}$$
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II M 3) Given the function $f(x, y) = x^2 - y^2 - 2xy^2 + 10x$, analyze the nature of its stationary points.

II M 4) Given $f(x, y) = (x + y)e^{x+y}$ and the unit vector $v = (\cos \alpha, \sin \alpha)$, determine the values for α for which the directional derivatives $\mathcal{D}_v f(1, 1)$ are equal to zero and with such values of α calculate $\mathcal{D}_{v,v}^2 f(1, 1)$.