UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2021/22
Quantitative Methods for Economic Applications - Mathematics for Economic Applications
Task 30/5/2022

I M 1) Given the complex number $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$; find the roots of order fourth of the complex number $w = z^2$.

I M 2) Find the eigenvalues of the matrix $\mathbb{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$; and for the eigenvalue with algebraic multiplicity equal two calculate a basis of its associated eigenspace.

I M 3) Given a linear map $F\colon \mathbb{R}^4 \to \mathbb{R}^3$, with $F(x_1, x_2, x_3, x_4) = (x_1 + x_2 - x_3 - x_4, x_1 + x_2 + x_3 + x_4, mx_1 + kx_2 + mx_3 + kx_4)$; study, by varing the parameters m and k, the dimensions of the Kernel and of the Immage of F, and find a basis for the Immage when the dimension of the Kernel is maximum.

I M 4) Vector $V \in \mathbb{R}^3$ has coordinates $(1,2,3)$ respect the basis $\mathcal{B} = \{(1,1,1),(1,1,0),(1,0,0)\}$. Find the coordinates of V respect the basis $\mathcal{B}' = \{(0,0,1),(0,1,1),(1,1,1)\}$.

II M 1) Given the equation $f(x,y) = (x+y)^2 \cdot log\left(x^2 + y^2 - 1\right) = 0$ satisfied at the point $(1,1)$, verify that with it an implicit function $y = y(x)$ can be defined and then calculate, for this implicit function, its first derivative.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = xy \\ \text{u.c.: } \begin{cases} x^2 + y^2 - 1 \le 0 \\ x + y + 1 \le 0 \end{cases} \end{cases}$.

II M 3) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 \\ \text{u.c.: } x^2 - y^2 = 3 \end{cases}$.

II M 4) Given $f(x,y) = (x-y)e^{x-y}$ and the unit vector $v = (cos\,\alpha, -sin\,\alpha)$, determine the values for α for which the directional derivatives $\mathcal{D}_v f(1,1)$ are equal to zero and with such values of α calculate $\mathcal{D}^2_{v,v} f(1,1)$.