UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2021/22
Quantitative Methods for Economic Applications - Mathematics for Economic Applications
Task 27/5/2022

I M 1) Given the two complex numbers $z_1 = e^{\frac{\pi}{4}i}$ and $z_2 = e^{-\frac{\pi}{4}i}$; calculate the complex number $w = z_1 + z_2$ and find the roots of order fourth of w.

I M 2) Find the eigenvalues of the matrix $\mathbb{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$; and study if the matrix is diagonalizable or not.

I M 3) Given a linear map $F: \mathbb{R}^4 \to \mathbb{R}^3$, we know that:
1. vectors $v_1 = (1, 0, 0)$ and $v_2 = (0, 0, 1)$ belong to the Immage of F;
2. vectors $w = (0, 1, 1, 0)$ belongs to the Kernel of F;
3. $F(1, 1, 1, 1) = (1, 1, 1)$.
Calculate the dimension of the Immage and the dimension of the Kernel, and for both the sets find a basis.

I M 4) Given the matrix $\mathbb{A} = \begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix}$ and knowing that its determinant is equal 20; find the value of the parameter a and calculate the inverse matrix of $\mathbb{A}$.

II M 1) Given the equation $f(x, y) = \dfrac{sin(x - y)}{x + y} = 0$ satisfied at the point $(1, 1)$, verify that with it an implicit function $y = y(x)$ can be defined and then calculate, for this implicit function, its first derivative.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x + y \\ \text{u.c.: } 0 \leq y \leq 1 - x^2 \end{cases}$.

II M 3) Check if the function $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ is differentiable at $(0, 0)$.

II M 4) Given the function $f(x, y) = x^2 + y^2 + 2x - 2y$, the two unit vectors $v = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2} \right)$ and $w = \left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2} \right)$, and the point $P = (x_P, y_P)$; if the two directional derivatives $\mathcal{D}_v f(P)$ and $\mathcal{D}_w f(P)$ are both equal to $\sqrt{2}$, find the coordinates of point P and calculate the second order directional derivative $\mathcal{D}^2_{v,w} f(P)$.