UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2021/22
Quantitative Methods for Economic Applications -
Mathematics for Economic Applications
Task 6/10/2022

I M 1) Solve the following equation: $x^3 + x^2 + x + 1 = 0$; and find the roots of order two of the unique solution with positive imaginary part.

I M 2) Given the matrix $\mathbb{A} = \begin{bmatrix} a & 1 & 1 \\ b & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and knowing that $(1, 1, 1)$ is an eigenvector associated to the eigenvalue $\lambda = 1$; find the value of the parameters a and b and calculate the value of the another eigenvalue of the matrix $\mathbb{A}$.

I M 3) Given the linear homogeneous system $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ mx_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + mx_2 + kx_3 + kx_4 = 0 \end{cases}$, find, on varying the parameters k and m, the dimension of the linear space of its solutions.

I M 4) Given a linear map $F: \mathbb{R}^2 \to \mathbb{R}^4$, we know that vectors $v_1 = (1, 0, 0, 0)$ and $v_2 = (0, 0, 0, 1)$ belong to the Image of F. Calculate the dimension of the Image and the dimension of the Kernel, and for the Image find a basis.

II M 1) Given the equation $f(x, y) = log(e + xy) - e^{x+y} = 0$ satisfied at the point $(0, 0)$, verify that with it an implicit function $y = y(x)$ can be defined and then calculate, for this implicit function, its first derivative.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x^2 + y \\ \text{u.c.: } \begin{cases} x^2 + y^2 \leq 1 \\ y \geq 0 \end{cases} \end{cases}$.

II M 3) Given the function $f(x, y, z) = x^2 + 4(x - 1)y^2 + 2yz$, analyze the nature of its stationary point.

II M 4) Given the function $f(x, y) = x\, e^{x+y} - y\, e^{x-y}$ and the unit vectors $v = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2} \right)$; find the two directional derivatives $\mathcal{D}_v f(0, 0)$ and $\mathcal{D}^2_{v,v} f(0, 0)$.