UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2022/23
Intermediate Test Quantitative Methods for Economic Applications - Mathematics (18/11/22)

1) Given the complex number $z = \frac{i(1+i)}{(1-i)^2}$. Calculate its square roots.

2) Consider the matrix: $\mathbb{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$. Calculate its eigenvalues and for any eigenvalue find a basis for its associated eigenspace. The matrix is diagonalizable?

3) Given a linear map $F: \mathbb{R}^3 \to \mathbb{R}^4$, with $F(x_1, x_2, x_3) = (x_2, x_1 + x_3, x_1 + x_2 + x_3, x_1 - x_2 + x_3)$. Calculate the dimention of its immage and the dimention of its kernel; and for both, immage and kernel, find a basis.

4) Consider a vector space V, and two vector subspaces of V, S and T. Prove that their intersection, $S \cap T$, is a vector subspaces of V.