UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2022/23
Quantitative Methods for Economic Applications - Mathematics for Economic Applications
Task 6/2/2023

I M 1) 1) Given the complex number $z = \left(\frac{1}{i^3} + \frac{1}{i^4}\right)^2$, calculate its cubic roots.

I M 2) Consider the matrix $\mathbb{A} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Calculate its eigenvalues and find the matrix P that diagonalizes $\mathbb{A}$.

I M 3) Given a linear map $F \colon \mathbb{R}^3 \to \mathbb{R}^3$, we know that:

1. $F(1,0,1) = F(0,1,0) = (0,0,0)$;
2. $F(0,0,1) = (0,0,1)$.

Find the dimention of its image and the dimention of its kernel; and for both, image and kernel, determine a basis.

I M 4) Given the linear system $\begin{cases} mx_1 + kx_2 + kx_3 = 6 \\ x_1 + mx_2 + kx_3 = 6 \\ x_1 + x_2 + mx_3 = 6 \end{cases}$, where m and k are two real parameters and knowing that the vector $(2,2,2)$ is a solution of it; find the values of m and k, and calculate the number of solutions of the linear system.

II M 1) Given the equation $f(x,y,z) = x(y+z) + e^{x+y+z} = 0$ satisfied at the point $(1,0,-1)$, verify that with it an implicit function $z = z(x,y)$ can be defined and then calculate, for this implicit function, its directional derivatives $\mathcal{D}_v z(1,0)$, where v is the unit vector $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 \\ \text{u.c.: } 0 \le y \le \frac{1}{4} - x^2 \end{cases}$.

II M 3) Check if the function $f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ is differentiable at point $(0,0)$.

II M 4) Given the function $f(x,y) = e^{xy}$ and the two unit vectors $v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $w = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, find all the points (x_0, y_0) such that $\mathcal{D}_v f(x_0,y_0) + \mathcal{D}_w f(x_0,y_0) = 0$.