

UNIVERSITA' DEGLI STUDI DI SIENA

Facoltà di Economia "R. Goodwin"

A.A. 2022/23

Quantitative Methods for Economic Applications -

Mathematics for Economic Applications

Task 29/5/2023

I M 1) Given the equation $x^2 - 2x + k = 0$, knowing that $x = 1 - i$ is a root of the equation, calculate the value of k and find the square roots of the other equation's root.

I M 2) Consider the matrix $\mathbb{A} = \begin{bmatrix} 5 & -1 & -3 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Calculate its eigenvalues and study if the

matrix \mathbb{A} is a diagonalizable one.

I M 3) Given a linear map $F: \mathbb{R}^4 \rightarrow \mathbb{R}^4$, with $F(x_1, x_2, x_3, x_4) = (x_1, x_2 + x_3, x_2 + x_3 + x_4, x_4)$, calculate the dimension of its image and the dimension of its kernel; and for both, image and kernel, determine a basis.

I M 4) Consider the matrix $\mathbb{U} = \begin{bmatrix} 1 & k \\ -k & -1 \end{bmatrix}$. Knowing that the matrix \mathbb{U} is a orthogonal matrix, calculate the value of k and find the matrix \mathbb{U}^2 .

II M 1) Given the equation $f(x, y) = x \sin y - y \cos x + x - y = 0$ satisfied at the point $(0, 0)$, verify that with it an implicit function $y = y(x)$ can be defined and then calculate, for this implicit function the first derivative $y'(0)$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x^2 + y^2 \\ \text{u.c.: } x^2 \leq y \leq 4 \end{cases}$.

II M 3) Check if the function $f(x, y) = \begin{cases} \frac{x^3 \cdot y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ is differentiable at point $(0, 0)$.

II M 4) Consider the two unit vectors $e_1 = (1, 0)$, $e_2 = (0, 1)$ and the function

$f(x, y) = ax^2 + bxy$. If $\mathcal{D}_{e_1} f(1, 1) = 1$ and $\mathcal{D}_{e_2} f(-1, -1) = -1$, find the values of a and b and calculate $\mathcal{D}_{e_1, e_2}^{(2)} f(1, -1)$.