UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
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Quantitative Methods for Economic Applications - Mathematics for Economic Applications
Task 26/6/2023

I M 1) z_1 and z_2 are two complex numbers, z_1 has module $\rho_1 = 1$ and argument $\alpha_1 = \frac{1}{4}\pi$, while z_2 has module $\rho_2 = 4$ and argument $\alpha_1 = \frac{5}{4}\pi$. Write in algebraic and goniometric form the complex number $w = z_1 \cdot z_2$, and find the square roots of w.

I M 2) Consider the matrix $\mathbb{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$. Calculate its eigenvalues and study if the matrix $\mathbb{A}$ is a diagonalizable one.

I M 3) Given a linear map $F\colon \mathbb{R}^3 \to \mathbb{R}^4$, we know that:

1. vectors $(1,0,0)$ and $(1,1,0)$ belong to the kernel of F;
2. the immage of vector $(1,1,1)$ is the vector $(1,0,1,0)$.

Find the matrix $\mathbb{A}_F$ associated with the linear map and calculate the dimention of the kernel and the dimention of the image of the linear map.

I M 4) Consider the matrix $\mathbb{U} = \begin{bmatrix} k & 1 \\ 1 & k \end{bmatrix}$. Knowing that the matrix $\mathbb{U}$ is a horthogonal matrix, calculate the value of k and find the eigenvalues of matrix $\mathbb{U}$.

II M 1) Given the equation $f(x,y,z) = e^{x+y+z} - xy - yz = 1$ satisfied at the point $(0,0,0)$, verify that with it an implicit function $z = z(x,y)$ can be defined and then calculate, for this implicit function the gradient ∇z.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 - y \\ \text{u.c.: } x^2 + y^2 \leq 4 \end{cases}$.

II M 3) Find, if it exist, the maximum and the minimum of the function $f(x,y,z) = x^2 + y^3 - 6y^2 + z^2$.

II M 4) Consider the function $f(x,y) = xe^{x+y}$ and the two unit vectors $e_1 = (1,0)$ and $e_2 = (0,1)$; at point (x_0, y_0) the two directional derivatives $\mathcal{D}_{e_1} f(x_0,y_0)$ and $\mathcal{D}_{e_2} f(x_0,y_0)$ are respectively equal to 0 and -1. Find the point (x_0,y_0) and calculate the second order directional derivative $\mathcal{D}^{(2)}_{e_1,e_2} f(x_0,y_0)$.