UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2022/23
Quantitative Methods for Economic Applications -
Mathematics for Economic Applications
Task 12/09/2023

I M 1) x^* is the unique solution of the equation $x^4 - 1 = 0$ with negative imaginary part; find the cubic roots of x^*.

I M 2) Consider the matrix $\mathbb{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$. Calculate its eigenvalues and the dimentions of their eigenspaces.

I M 3) Given a linear map $F\colon \mathbb{R}^3 \to \mathbb{R}^3$, with $F(x_1, x_2, x_3) = (x_1 + x_2 + x_3, kx_1 + kx_2 + mx_3, x_2 + mx_3)$, where k and m are real parameters; we know that the immage of vector $(1, 1, 1)$ is the vector $(3, 3, 2)$. Find the value of the two parameters k and m and calculate a basis for the immage of such linear map.

I M 4) Consider the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 3 & 3 & 0 \\ 4 & 4 & 4 & 4 \end{bmatrix}$. Is this matrix a diagonalizable one? Justify the answer.

II M 1) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x^2 - 3\,y \\ \text{u.c.:} \begin{cases} x^2 + y^2 \le 4 \\ x - y \le 0 \end{cases} \end{cases}$.

II M 2) Given the system of equations $\begin{cases} f(x, y, z) = x + y + z = 1 \\ g(x, y, z) = x^2 + y^2 + z^2 = 3 \end{cases}$, satisfied at the point $(1, -1, 1)$, verify that with it an implicit function $z \to (x(z), y(z))$ can be defined and then calculate, for this implicit function, the derivatives $x'(1)$ and $y'(1)$.

II M 3) Find, if it exists, the maximum and the minimum of the function $f(x, y) = x^3 + y^3 - xy$.

II M 4) Consider the function $f(x, y, z) = xy\, e^{xy+z}$ and the unit vector $v = (0, 1, 0)$; calculate the first and second order directional derivateves $\mathcal{D}_v f(P)$ and $\mathcal{D}^{(2)}_{v,v} f(P)$, where P is the point $(1, 0, 1)$.