UNIVERSITA' DEGLI STUDI DI SIENA
Facoltà di Economia "R. Goodwin"
A.A. 2023/24
Intermediate Test Quantitative Methods for Economic Applications - Mathematics (27/11/23)

1) Given the complex number $z = \frac{1}{1+i} - \frac{1}{1-i}$. Calculate its cubic roots.

2) Consider the matrix: $\mathbb{A} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ k & -1 & 1 \end{bmatrix}$. We know that the characteristics polynomial of the matrix $\mathbb{A}$, $p_{\mathbb{A}}(\lambda) = \lambda(\lambda - 2)^2$. Find the value of parameter k, and for the two eigenvalues of matrix $\mathbb{A}$ calculate the dimention of their eigenspaces.

3) Given a linear map $F: \mathbb{R}^4 \to \mathbb{R}^4$, with $F(x_1, x_2, x_3, x_4) = (x_1, x_1, x_4, x_4)$. Calculate the dimention of its immage and the dimention of its kernel; and for both, immage and kernel, find a basis.

4) Calculate the inverse matrix of matrix $\mathbb{B} = \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & \frac{1}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$.

5) Consider a matrix $\mathbb{A}$, we know that λ is an eigenvalue of the matrix $\mathbb{A}$ with associated eigenvector $\boldsymbol{v}$; prove that λ^2 is an eigenvalue of the matrix $\mathbb{A}^2$ with associated the same eigenvector $\boldsymbol{v}$.