UNIVERSITA' DEGLI STUDI DI SIENA Facoltà di Economia "R. Goodwin" A.A. 2022/23 Quantitative Methods for Economic Applications - Mathematics for Economic Applications Task 18/3/2024

IVEN TATE: The COLORGIBLE TASK 18/3/2024

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coloulet **Task 18/3/2024**
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4; the request complex numbers are $z_1 = 4 + 2i$ and $z_2 = -4 + 2i$. For

s reme odule $\rho = \sqrt{a^2 + 9}$. Put $\sqrt{a^2 + 9} = 5$ follow $a^2 + 9 = 25$ and $a^2 = 16 \pm 4$; the request complex numbers are $z_1 = 4 + 2i$ and $z_2 = -4 + 2i$. For this remember that if $a \neq 0$, the argument of z can be calculated by its arguments remember that if $a \neq 0$, the argument of z can be calculated by

with the module
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. Put $\sqrt{a^2 + 9} = 3$ follow $a + 9 = 23$ and $a =$
with $a = \pm 4$; the request complex numbers are $z_1 = 4 + 2i$ and $z_2 = -4 + 2i$
its arguments remember that if $a \neq 0$, the argument of z can be calculated by
 $\theta = arctg\left(\frac{b}{a}\right)$ and for z_1 and z_2 we get $\theta_{1,2} = arctg\left(\pm \frac{2}{4}\right) = \pm arctg\left(\frac{1}{2}\right)$;
 $\theta_{1,2} \approx \pm 0.46$ radiant.
IM 2) Given the matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 1 \\ 3 & k & -2 \end{bmatrix}$ and knowing that 1 is an eigenval

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\nIM 2) Given the matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 1 \\ 3 & k & -2 \end{bmatrix}$ and knowing that 1 is an eigenvalue of the matrix; study if the matrix A is diagonalizable or not.

\nAt the first step we calculate the characteristic polynomial of matrix A ;

\n
$$
P_A(\lambda) = |\lambda \mathbb{I} - A| = \begin{vmatrix} \lambda - 2 & -2 & 1 \\ 1 & \lambda - 2 & -1 \\ 2 & \lambda - 2 & -1 \\ 3 & \lambda - 2 & -1 \end{vmatrix} = (\lambda - 2) \cdot \begin{vmatrix} \lambda - 2 & -1 \\ -k & \lambda + 2 \end{vmatrix} +
$$

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 $P_A(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda - 2 & -2 & 1 \\ 1 & \lambda - 2 & -1 \\ -3 & -k & \lambda + 2 \end{vmatrix} = (\lambda - 2) \cdot \begin{vmatrix} \lambda - 2 & -1 \\ -k & \lambda + 2 \end{vmatrix} +$
 $- 2 \cdot \begin{vmatrix} 1 & -1 \\ -3 & \lambda + 2 \end{vmatrix} - \begin{vmatrix} 1 & \lambda - 2 \\ -3 & -k \end{vmatrix} = (\lambda - 2) \cdot ((\lambda - 2)(\lambda + 2) - k) +$
 $- 2 \cdot (\lambda + 2 - 3) - (-k + 3(\lambda - 2)) = (\lambda - 2) \cdot (\lambda^2 - 4 - k) - 2 \cdot (\lambda - 1) +$
 $- (3\lambda - 6 - k) = \lambda^3 - 2\lambda^2 - (9 + k)\lambda + 16 + 3k$. If 1 is an eigenvalue of the matrix,
1 is a root of the characteristic polynomial thus $P_A(1) = 0$ and $P_A(1) = 6 + 2k$; put
 $6 + 2k = 0$ easily we find $k = -3$. Matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 1 \\ 3 & -3 & -2 \end{bmatrix}$ and
 $P_A(\lambda) = \lambda^3 - 2\lambda^2 - 6\lambda + 7 = (\lambda - 1) \cdot (\lambda^2 - \lambda - 7)$.
Now we calculate the remaining two eigenvalues putting $\lambda^2 - \lambda - 7 = 0$, the equation
has solutions $\frac{1 \pm \sqrt{29}}{2}$ and the three eigenvalues of A are $\lambda_1 = 1$ and
 $\lambda_{2,3} = \frac{1 \pm \sqrt{29}}{2}$; matrix is diagonalizable because its three eigenvalues are one to one
distinct.
1 M 3) Given the linear system $\begin{cases} mx_1 + mx_2 + mx_3 = 0 \\ mx_1 + mx_2 + x_3 = 0 \end{cases}$, where m is a real

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$$
, where *m* is a real
parameter. We indicate with S_m the set of its solutions, study, varing of *m*, the
dimension of the set S_m , and when the dimension is bigger, find a basis for S_m .

The matrix associated to the system is $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$. We reduce the matrix by elementary $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$. We reduce the map $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$. We reduce the $\begin{pmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{pmatrix}$. We reduce $1 \mid$. We re $\begin{bmatrix} m & m \\ m & 1 \\ 1 & 1 \end{bmatrix}$. We reduce the r . We reduce

operations on its lines: The matrix associated to the system is $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$. We reduce the matrix by elementary
operations on its lines:
 $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix} \begin{bmatrix} R_2 \mapsto R_2 - R_1 \\ R_3 \mapsto R_3 - R_1 \end{bmatrix} \begin{bmatrix} m & m & m \\ 0$ operations on its lines:
 $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$
 $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$
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 $\begin{bmatrix} m & m & m \\ 0 & 0 & 1 - m \\ 0 & 1 - m & 1 - m \end{bmatrix}$
 $\begin{bmatrix} m & m & m \\ 0 & 0 & 1 - m \\ 0 & 0 & 1 - m \$ ne matrix associated to the system is $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$. We reduce the matrix by elementary
perations on its lines:
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 $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$ $\begin{bmatrix} R_2 \mapsto R_2 - R_1 \\ R_3 \mapsto R_3 - R_1 \end{bmatrix}$ $\begin{bmatrix} m & m & m \\ 0 & 0 & 1$ e matrix associated to the system is $\begin{bmatrix} m & m & m & m \\ m & m & 1 & 1 \ m & 1 & 1 \end{bmatrix}$. We reduce the matrix by elementary

erations on its lines:
 $\begin{bmatrix} m & m & m \\ m & m & 1 \ m & 1 & 1 \end{bmatrix} \begin{bmatrix} R_2 \mapsto R_2 - R_1 \\ R_3 \mapsto R_3 - R_1 \end{bmatrix} \begin{bmatrix} m & m & m$ s lines:
 $R_2 \mapsto R_2 - R_1$
 $R_3 \mapsto R_3 - R_1$
 $0 \qquad 0 \qquad 1 - m$
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 $0 \qquad 0 \qquad 0$ operations on its lines:
 $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix} \begin{bmatrix} R_2 \mapsto R_2 - R_1 \\ R_3 \mapsto R_3 - R_1 \end{bmatrix} \begin{bmatrix} m & m & m \\ 0 & 0 & 1 - m \\ 0 & 1 - m & 1 - m \end{bmatrix} \begin{bmatrix} m & m & m \\ 0 & 1 - m & 1 - m \\ 0 & 0 & 1 - m \end{bmatrix}$

The determinant of the reduced matrix $\begin{bmatrix} n & m \\ m & 1-m \\ 1-m \end{bmatrix}$.
 $(1-m)^2$ and it is different
 m] $\begin{bmatrix} m \ 1-m \ 1-m \end{bmatrix} R_2 \circ R_3 \begin{bmatrix} m & m & m \ 0 & 1-m & 1-m \ 0 & 0 & 1-m \end{bmatrix}$
 $\begin{bmatrix} m & m & m \ 0 & 1-m & 1-m \ 0 & 0 & 1-m \end{bmatrix} = m(1-m)^2$ $\begin{bmatrix} m \ 1-m \ 1-m \end{bmatrix}$ $R_2 \circ R_3$ $\begin{bmatrix} m & m & m \ 0 & 1-m & 1-m \ 0 & 0 & 1-m \end{bmatrix}$.
 $\begin{bmatrix} m & m \ m & m & m \ 0 & 1-m & 1-m \ 0 & 0 & 1-m \end{bmatrix}$ = $m(1-m)^2$ and it $\begin{bmatrix} 1-m \ 1-m \end{bmatrix}$ $R_2 \circ R_3$ $\begin{bmatrix} m & m & m \ 0 & 1-m & 1-m \ 0 & 0 & 1-m \end{bmatrix}$
 $\begin{bmatrix} m & m \ 0 & 1-m \end{bmatrix}$ $= m(1-m)^2$ and
 $\begin{bmatrix} m & m & m \ 0 & 1-m \end{bmatrix}$. $\begin{bmatrix} m & m \\ 1-m & 1-m \\ 0 & 1-m \end{bmatrix}$.
= $m(1-m)^2$ and it is different and it is different The determinant of the reduced matrix is $\begin{vmatrix} m & m & m \\ 0 & 1-m & 1-m \\ 0 & 0 & 1-m \end{vmatrix} = m(1-m)^2$ and it is different
from zero if and only if $m \neq 0$ and $m \neq 1$, rank of matrix $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$ is three if an $\begin{bmatrix} 0 & 0 & 1-m \end{bmatrix}$
 $\begin{bmatrix} m \\ m \end{bmatrix} = m(1-m)^2$ and it is diff
 $\begin{bmatrix} m & m & m \\ m & 1 & 1 \\ m & 1 & 1 \end{bmatrix}$ is three if and of and $m \neq 1$, rank of matrix $\begin{bmatrix} m & m & m \\ 0 & 1-m & 1-m \\ 0 & 0 & 1-m \end{bmatrix} = m(1-m)^2$ and it is different and $m \neq 1$, rank of matrix $\begin{bmatrix} m & m & m \\ m & 1 & 1 \\ m & 1 & 1 \end{bmatrix}$ is three if and only from zero if and only if $m \neq 0$ and $m \neq 1$, rank of matrix $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$ is three if and only if $m \neq 0$ and $m \neq 1$ and this imply that in this case the dimention of S_m is zero. If $m = 1$ the from zero if and only if $m \neq 0$ and $m \neq 1$, rank of matrix $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$ is three if and only if $m \neq 0$ and $m \neq 1$ and this imply that in this case the dimention of S_m is zero. If $m = 1$ the imply that in this case the dimensional point
 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, with rank equal at

red to the system is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 0 and $m \neq 1$, rank of matrix

mply that in this case the dim
 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, with rank equal 0 and $m \neq 1$, rank of matrix $\begin{bmatrix} m & m & m \\ m & m & 1 \\ m & 1 & 1 \end{bmatrix}$ is three if and only if

mply that in this case the dimention of S_m is zero. If $m = 1$ the matrix
 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, with rank eq or $m \ne 1$, take of matrix
 y that in this case the dim
 $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, with rank equal
 $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $m \neq 0$ and $m \neq 1$ and this imply that in this case the dimention of S_m is zero. If $m = 1$ the matrix associated to the system is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, with rank equal at 1 and dimention of S_m 2 se the dimention of S_m is ze
ank equal at 1 and dimention
0 0 0 1
0 with rank equal a rank equal at 1 and dimention of
 $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, with rank equal at
 $= 1$
 $= 0$ Dimention of S is bigger , with rank equal at 2 and dimentic $m = 0$ the matrix associated to the system is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, with rank equal at 2 and dimention of
 S_m 1. In conclusion $dim(S_m) = \begin{cases} 2 & \text{if } m = 1 \\ 1 & \text{if } m = 0 \\ 0 & \text{otherwise} \end{cases}$. Dimention of S_m i $m=1$ $n = 0$ the matrix associated to the system is $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, with rank equal at 2 and d
 n_m 1. In conclusion $dim(S_m) = \begin{cases} 2 & \text{if } m = 1 \\ 1 & \text{if } m = 0 \\ 0 & \text{otherwise} \end{cases}$. Dimention of S_m is bigger if $m = 0$ if $m =$ system is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, with rank
 $2 \text{ if } m = 1$
 $1 \text{ if } m = 0$. Dimention of S_m the system is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, with rank equal at 2 and dimention of 2 if $m = 1$
1 if $m = 0$. Dimention of S_m is bigger if $m = 1$, and in this 0 otherwise 0 otherw $\frac{1}{2}$ if $m = 1$ $\{1 \text{ if } m=0.1 \}$ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ otherwise S_m 1. In conclusion $dim(S_m) = \begin{cases} 2 & \text{if } m = 1 \\ 1 & \text{if } m = 0 \end{cases}$. Dimention of S_m is bigger if $m = 1$, and in this
case the system is reduced to the unique equazion $x_1 + x_2 + x_3 = 0$ or $x_3 = -x_1 - x_2$, and a
generic ele S_m 1. In conclusion $dim(S_m) = \begin{cases} 2 & \text{if } m = 1 \\ 1 & \text{if } m = 0 \end{cases}$. Dimention of S_m is bigger if $m = 1$, and in this
case the system is reduced to the unique equazion $x_1 + x_2 + x_3 = 0$ or $x_3 = -x_1 - x_2$, and a
generic ele S_m 1. In conclusion $dim(S_m) = \begin{cases} 2 & n \le -1 \\ 1 & \text{if } m = 0 \text{ .} \text{ Dimension of } S_m \text{ is bigger if } m = 1 \text{, and in this} \\ 0 & \text{otherwise} \end{cases}$
case the system is reduced to the unique equazion $x_1 + x_2 + x_3 = 0$ or $x_3 = -x_1 - x_2$, and a generic element of S_1 is $\begin{cases}\n\text{or } x_m \text{ is reduced to the unique equation } x_1 + x_2 + x_3 = 0 \text{ or } x_3 = -x_1 - x_2 \text{, and a generic element of } S_1 \text{ is } (x_1, x_2, -x_1 - x_2) = x_1(1, 0, -1) + x_2(0, 1, -1), \text{ a basis for } S_1 \text{ is the set of vectors } S_{S_1} = \{(1, 0, -1), (0, 1, -1)\}. \text{ To be thorough we consider also the case } m = 0, \text{ in this situation the system is reduced to } \begin{cases}\n x_3 = 0 \\
 x_2 + x_3 = 0\n \end{cases}\$ azion $x_1 + x_2 + x_3 = 0$ or $x_3 = -$
= $x_1(1, 0, -1) + x_2(0, 1, -1),$
 $\}$. To be thorough we consider als
 $x_3 = 0$
 $x_2 + x_3 = 0$ \Rightarrow $\begin{cases} x_3 = 0 \\ x_2 = 0 \end{cases}$, and a g = $x_1(1, 0, -1) + x_2(0, 1, -1),$
 $\}$. To be thorough we consider als
 $x_3 = 0$
 $x_2 + x_3 = 0$ \Rightarrow $\begin{cases} x_3 = 0 \\ x_2 = 0 \end{cases}$, and a g
 x_3 is the set $\mathcal{B}_{S_0} = \{(1, 0, 0)\}$. case the system is retated to the unique equation $x_1 + x_2 + x_3 = 0$ or $x_3 = -x_1 - x_2$, generic element of S_1 is $(x_1, x_2, -x_1 - x_2) = x_1(1, 0, -1) + x_2(0, 1, -1)$, a basis for set of vectors $B_{S_1} = \{(1, 0, -1), (0, 1, -1)\}$. To in this situation the system is reduced to $\begin{cases} x_3 = 0 \\ x_2 + x_3 \end{cases}$

S₀ is $(x_1, 0, 0) = x_1(1, 0, 0)$, a basis for S_0 is the sequence of $\begin{cases} x_3 = 0 \\ x_2 + x_3 \end{cases}$

I M 4) Given a linear map $F: \mathbb{R}^3 \to \mathbb{R}^3$, in this situation the system is reduced to $\begin{cases} x_3 = 0 \\ x_2 + x_3 \end{cases}$
 S_0 is $(x_1, 0, 0) = x_1(1, 0, 0)$, a basis for S_0 is the se IM 4) Given a linear map $F: \mathbb{R}^3 \to \mathbb{R}^3$, we know t
 $1. F(1, 1, 1) = (0, 0, 0);$

S₀ is $(x_1, 0, 0) = x_1(1, 0, 0)$, a basis for S_0 is the set $\mathcal{B}_{S_0} = \{(1, 0, 0)\}$.
 IM4) Given a linear map $F: \mathbb{R}^3 \to \mathbb{R}^3$, we know that:

1. $F(1, 1, 1) = (0, 0, 0)$;

2. $F(0, 0, 1) = (0, 0, 1)$;

3. $F(1,$

$$
M(4)
$$
 Given a linear map $F: \mathbb{R}^3 \to \mathbb{R}^3$, we know that:

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Find the dimention of its image and the dimention of its kernel; and for both, image and kernel, set a basis.

For the belongs in the belongs in the the belongs in the kernel; and for both, image and kernel, set
a basis.
For the linear map F we know that the vector $(1, 1, 1)$ belongs in the kernel of F, thus
the dimention of the 2. $F(0, 0, 1) = (0, 0, 1)$,

3. $F(1, 0, 0) = (1, 0, 0)$.

Find the dimention of its image and the dimention of its kernel; and for both, image and kerne

a basis.

For the linear map F we know that the vector $(1, 1, 1)$ be S. $F(1,0,0) = (1,0,0)$.
Find the dimention of its image and the dimention of its kernel; and for both, image and kernel, a basis.
For the linear map F we know that the vector $(1,1,1)$ belongs in the kernel of F, thus
the For the linear map F we know that the vector $(1, 1, 1)$ belongs in the kernel of F, thus
the dimention of the kernel is at least one: $dim(KerF) \ge 1$. The two linear indipendent
vectors $(0, 0, 1)$ and $(1, 0, 0)$ belong in inequalities easily we conclude that $dim(KerF) = 1$ and $dim(ImaF) = 2$. The two For the finear linap T we know that the vector $(1, 1, 1)$ belongs in the kerifier of T , thus
the dimention of the kernel is at least one: $dim(KerF) \ge 1$. The two linear indipendent
vectors $(0, 0, 1)$ and $(1, 0, 0)$ be the unifiable of the senier is at least one. $\lim_{k \to \infty} \frac{km(k + kT)}{2} \ge 1$. The two fined imappendent
vectors (0, 0, 1) and (1, 0, 0) belong in the image of F, thus the dimention of the image
is at least two: $\dim(\text{Im}F) \ge$ vectors $(0, 0, 1)$ and $(1, 0, 0)$ belong in the finage of P , thus the dimention of the linear
is at least two: $dim(ImaF) \ge 2$. By the dimention Theorem is known that for the linear
map F , $dim(KerF) + dim(ImaF) = dim(\mathbb{R}^3) = 3$,

basis for the spaces kernel and time ($KerY$) = 1 and $dim(ImY)$ = 2. The two
basis for the spaces kernel and image can be easily found as $B_{KerF} = \{(1, 1, 1)\}$ and
 $B_{ImAF} = \{(0, 0, 1), (1, 0, 0)\}$.
II M 1) The equation $f(x, y) = 0$ If M 1) The equation $f(x, y) = 0$ satisfied on point $P(x_P, y_P)$, defined an implicit
function $y = y(x)$. We know that the gradient of f on point P is $\nabla f(P) = (1, -1)$
and the Hessian matrix of f on point P is $\mathcal{H}f(P) = \begin{bmatrix}$

By the Dini's Theorem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; and
 $f''_{xx}(P) + 2 \cdot f''_{xx}(P) \cdot y'(x_P) + f''_{xx}(P) \cdot (y'(x_P))^2$ $\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$;
 $\frac{(P) \cdot (y'(x_P))^2}{(P) \cdot (y'(x_P))^2} =$ $y'(P) \neq 0, y'(x_P) = -\frac{f'_x(P)}{f'(P)} =$ $y'(P)$ $(P) \neq 0, y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; and
 $\frac{f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2}{(x'(P))^2} =$ By the Dini's Theorem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; and
 $y''(x_P) = -\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2}{(f'_y(P))^2} =$
 $f''(P) + 2 \cdot f''(P) + f''(P)$ Theorem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; and
 $\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2}{(f'_y(P))^2} =$ $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} =$
 $y'(x_P) + f''_{y,y}(P) \cdot (y'(f'_y(P)))^2$
 $\frac{f'_y(P)}{f'_y(y(P))} =$ $^{\prime\prime}$ ($_{r}$, neorem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$
 $f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2 =$
 $(f'_x(P))^2$ $\overline{2}$ $\left(y'(P)\right)^2$ the Dini's Theorem if f'_y
 $(x_P) = -\frac{f''_{x,x}(P) + 2 \cdot f''_{x,x}(P) + 2 \cdot f'''_{x,x}(P)}{f'''_{x,x}(P) + 2 \cdot f'''_{x,x}(P)}$ orem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; and
 $\frac{(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2}{(f'_y(P))^2} =$ x_P) = - $\frac{3}{2}$ x_P) = $-\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; and
 $\frac{x_P$) + $f''_{y,y}(P) \cdot (y'(x_P))^2}{(P))^2} =$ e Dini's Theorem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{1}{-1} = 1$; a
 $F''(P) = -\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2}{(f'_y(P))^2} = -\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)}{(f'_y(P))^2} = -\frac{(f'_y(P))^2}{1 + 2 \cdot (-1) + 1}$ Theorem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{f'_x(P)}{f'_y(P)} = -\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))}{(f'_y(P))^2}$
 $\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)}{(f'_y(P))^2} = \frac{f''_{x,y}(P) + f''_{y,y}(P)}{(f'_y(P))^2}$ $\begin{split} &\mathcal{L}_{x,x}''(P)+2\cdot f''_{x,y}(P)\cdot y'(x_P)+f''_{y,y}(P)\ &\qquad \qquad \left(f'_y(P)\right)^2\ &\qquad \qquad \left(f'_y(P)\right)^2\ &\qquad \qquad \left(f'_y(P)+f''_{y,y}(P)\right)=\ &\qquad \qquad \left(f'_y(P)\right)^2\ &+2\cdot(-1)+1\ &\qquad \qquad \qquad \left(f'_y(P)\right)^2=0\,. \end{split}$ $-\frac{f''_{x,x}(P)+2\cdot f''_{x,y}(P)+f''_{y,y}(P)}{\left(f'_y(P)\right)^2}=$ orem if $f'_y(P) \neq 0$, $y'(x_P) = -\frac{xy-1}{f'_y(P)} = -\frac{y}{f'_y(P)}$
 $\frac{(P) + 2 \cdot f''_{x,y}(P) \cdot y'(x_P) + f''_{y,y}(P) \cdot (y'(x_P))^2}{(f'_y(P))^2}$
 $\frac{(P) + 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)}{(f'_y(P))^2} =$ $\frac{2 \cdot f_{x,y}''(P) \cdot y'(x_P) + f_{y,y}''(P)}{\left(f_y'(P)\right)^2} \ = \ \frac{2 \cdot f_{x,y}''(P) + f_{y,y}''(P)}{\left(f_y'(P)\right)^2} \ = \ \frac{1) + 1}{2} = 0 \, .$ $-\frac{f_{x,x}(P) + 2 \cdot f_{x,y}(P) \cdot y (x_P) + f_{y,y}(P) \cdot y}{(f'_y(P))^2}$
 $-\frac{f''_{x,x}(P) + 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)}{(f'_y(P))^2} =$
 $-\frac{1 + 2 \cdot (-1) + 1}{(-1)^2} = 0.$ $(P) + 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)$
 $(f'_y(P))^2$
 $\frac{(\cdot(-1)+1)}{(-1)^2} = 0.$

oblem $\begin{cases} \text{Max/min } f(x, y) \\ \text{u.c.: } x^2 + y^2 \leq 0. \end{cases}$ $+ 2 \cdot f_{x,y}(P) \cdot y (x_P) + (f'_y(P))^2$
 $+ 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)$
 $(f'_y(P))^2$
 $-1) + 1$
 $(1)^2$ $(f'_y(P))$
 $)+ 2 \cdot f''_{x,y}(P) + f''_{y,y}(P)$
 $(f'_y(P))^2$
 $(-1) + 1$
 $(-1)^2 = 0$.

blem $\begin{cases} \text{Max/min } f(x, y) \end{cases}$. $f''_{y,y}(P)$
 $f(x,y) = x^3 + y^3$
 $y^2 \le 8$

toos function, the admissible region is 2
= 0.
 $\sqrt{\min f(x, y)} = x^3 + y^3$.
 $x^2 + y^2 \le 8$
ontinuos function, the admissible regional and alocad set constraint is qualify 0.
 $\sin f(x, y) = x^3 + y^2$
 $\frac{y^2 \le 8}{x}$
 $\sin \theta$ function the

II M 2) Solve the problem
$$
\begin{cases} \text{Max/min } f(x, y) = x^3 + y^3 \\ \text{u.c.: } x^2 + y^2 \le 8 \end{cases}
$$

The function f is a polynomial, continuos function, the admissible region is the interior region of a circunference, a bounded and closed set; constraint is qualified on circunference, therefore f presents absolute maximum and minimum in the admissible region. The Lagrangian function is The function f is a polynomial, continuos function, the adm
region of a circunference, a bounded and closed set; constra
circunference, therefore f presents absolute maximum and r
region. The Lagrangian function is
 $\mathcal{L$

The function *j* is a polynomial, continuous function, the admissible region of a circumference, a bounded and closed set; constraint is qualified on
circunference, therefore *f* presents absolute maximum and minimum in the admissible
region. The Lagrangian function is

$$
\mathcal{L}(x, y, \lambda) = x^3 + y^3 - \lambda(x^2 + y^2 - 8)
$$
 with

$$
\nabla \mathcal{L} = (3x^2 - 2\lambda x, 3y^2 - 2\lambda y, -(x^2 + y^2 - 8)).
$$

$$
\int_0^{\infty} \mathcal{L} = (3x^2 - 2\lambda x, 3y^2 - 2\lambda y, -(x^2 + y^2 - 8)).
$$

$$
\int_0^{\infty} \lambda = 0
$$

$$
3x^2 = 0
$$

$$
\int_0^{\infty} \lambda = 0
$$

$$
3y^2 = 0
$$

$$
\int_0^{\infty} \lambda = 0
$$

$$
y = 0
$$
; point (0, 0) is admissible, $\mathcal{H}f = \begin{bmatrix} 6x & 0 \\ 0 & 6y \end{bmatrix}$ and

$$
x^2 + y^2 - 8 \le 0
$$

$$
\mathcal{H}f(0, 0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{H}_2(0, 0) = 0
$$
. We haven't any information about the nature of
point (0, 0).

$$
\int_0^{\infty} \mathcal{L}f(0, 0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{H}_2(0, 0) = 0
$$
. We haven't any information about the nature of
point $(0, 0)$.

$$
\int_0^{\infty} \lambda \neq 0
$$

$$
\begin{cases}\n\begin{aligned}\n\frac{3y-6}{x^2+y^2-8} &\leq 0 \\
\frac{10}{x^2+y^2-8} &\leq 0\n\end{aligned} \\
\frac{4f(0,0)}{x^2-y^2-8} &\leq 0\n\end{cases}
$$
\n
$$
\begin{cases}\n\begin{aligned}\n\frac{10}{x^2-y^2-8} &\leq 0 \\
\frac{10}{x^2-y^2-8} &\leq 0\n\end{aligned} \\
\frac{10}{x^2-y^2-8} &\leq 0 \\
\frac{3x^2-2\lambda x}{3x^2-2\lambda x} &= 0 \\
\frac{3x^2-2\lambda y}{3x^2-2\lambda y} &= 0\n\end{aligned} \\
\frac{5x}{x^2+y^2-8} &= 0 \\
\frac{5x}{x^2+y^2-8} &\leq 0\n\end{cases}
$$
\n
$$
\begin{aligned}\n\frac{5y}{x^2+y^2-8} &= 0 \\
\frac{5y}{x^2+y^2-8} &\leq 0 \\
\frac{5y}{x^2+y^2-8} &\leq 0 \\
\frac{5y}{x^2+y^2-8} &\leq 0\n\end{aligned}
$$
\n
$$
\begin{aligned}\n\text{or } \text{must evaluate four possibilities:} \\
\frac{5y}{x^2+y^2-8} &\leq 0 \\
\frac{5y}{x^2+y^
$$

 $\int_{\alpha}^{2} x^2 + y^2 - 8 = 0$ $\int_{\alpha}^{2} x^2 + y^2 = 8$

a: if $x = 0$ and $y = 0$, $0^2 + 0^2 \neq 8$; point $(0, 0)$ isn't admissible;

b: if $x = 0$ and $y = \frac{2}{3}\lambda$, we get $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda^2 = 18 \Rightarrow \lambda = \pm 3\sqrt{2}$; point
 $P_1(0, 2\sqrt$ minimum; b: if $x = 0$ and $y = \frac{2}{3}\lambda$, we get $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda^2 = 18 \Rightarrow \lambda = \pm 3\sqrt{2}$; point
 $P_1(0, 2\sqrt{2})$ is a candidate to maximum, while point $P_2(0, -2\sqrt{2})$ is a candidate to

minimum;

c: if $y = 0$ and $x = \frac{2}{3}\lambda$, we g : if $x = 0$ and $y = \frac{2}{3}\lambda$, we get $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda^2 = 18 \Rightarrow \lambda = \pm 3\sqrt{2}$; point
 $P_1(0, 2\sqrt{2})$ is a candidate to maximum, while point $P_2(0, -2\sqrt{2})$ is a candidate to

inimum;

: if $y = 0$ and $x = \frac{2}{3}\lambda$, we get

 $P_1(0, 2\sqrt{2})$ is a candidate to maximum, while point $P_2(0, -2\sqrt{2})$ is a candidate to
minimum;
c: if $y = 0$ and $x = \frac{2}{3}\lambda$, we get again $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda = \pm 3\sqrt{2}$; point $P_3(2\sqrt{2}, 0)$
a candidate to maximum, wh minimum;

c: if $y = 0$ and $x = \frac{2}{3}\lambda$, we get again $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda = \pm 3\sqrt{2}$; point $P_3(2\sqrt{2}, 0)$ is

a candidate to maximum, while point $P_4(-2\sqrt{2}, 0)$ is a candidate to minimum;

d: if $x = \frac{2}{3}\lambda$ and $y = \frac{2$ and $x = \frac{2}{3}\lambda$, we get again $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda =$

e to maximum, while point $P_4(-2\sqrt{2}, 0)$ is a
 $\frac{2}{3}\lambda$ and $y = \frac{2}{3}\lambda$, we get $\frac{8}{9}\lambda^2 = 8 \Rightarrow \lambda^2 = 9$

e to maximum, while point $P_6(-2, -2)$ is a call if $y = 0$ and $x = \frac{2}{3}\lambda$, we get again $\frac{4}{9}\lambda^2 = 8 \Rightarrow \lambda = \pm 3\sqrt{2}$; point $P_3(2\sqrt{2}, 0)$ is
candidate to maximum, while point $P_4(-2\sqrt{2}, 0)$ is a candidate to minimum;
: if $x = \frac{2}{3}\lambda$ and $y = \frac{2}{3}\lambda$, we get $\frac{$ $\sqrt{2}$, 0) is
 $\binom{3}{5}(2,2)$ is c: if $y = 0$ and $x = \frac{1}{3}\lambda$, we get again $\frac{1}{9}\lambda^2 = 8 \Rightarrow \lambda = \pm 3\sqrt{2}$; point $P_3\left(\frac{2\sqrt{2}}{2}\right)$,
a candidate to maximum, while point $P_4\left(-\frac{2\sqrt{2}}{2}, 0\right)$ is a candidate to minimum;
d: if $x = \frac{2}{3}\lambda$ and $y = \frac$ a candidate to maximum, while point $P_4(-2\sqrt{2}, 0)$ is a candidate to minimum;

d: if $x = \frac{2}{3}\lambda$ and $y = \frac{2}{3}\lambda$, we get $\frac{8}{9}\lambda^2 = 8 \Rightarrow \lambda^2 = 9 \Rightarrow \lambda = \pm 3$; point $P_5(2, 2)$ is

a candidate to maximum, while point P_6 d: if $x = \frac{2}{3}\lambda$ and $y = \frac{2}{3}\lambda$, we get $\frac{8}{9}\lambda^2 = 8 \Rightarrow \lambda^2 = 9 \Rightarrow \lambda = \pm 3$; point $P_5(2, 2)$ is
a candidate to maximum, while point $P_6(-2, -2)$ is a candidate to minimum.
 $f(P_{1,3}) = 16\sqrt{2}$, $f(P_5) = 16 < 16\sqrt{2}$, f a candidate to maximum, while point $P_6(-2, -2)$ is a candidate to minimum.
 $f(P_{1,3}) = 16\sqrt{2}$, $f(P_5) = 16 < 16\sqrt{2}$, f presents absolute maximum equal $16\sqrt{2}$ on

points $(0, 2\sqrt{2})$ and $(2\sqrt{2}, 0)$; $f(P_{2,4}) = -16\sqrt{$ $f(P_{1,3}) = 16\sqrt{2}$, $f(P_5) = 16 < 16\sqrt{2}$, f presents absolute maximum equal $16\sqrt{2}$ on
points $(0, 2\sqrt{2})$ and $(2\sqrt{2}, 0)$; $f(P_{2,4}) = -16\sqrt{2}$, $f(P_6) = -16 > -16\sqrt{2}$, f
presents absoluteminimum equal $-16\sqrt{2}$ on p lower border of the admissible region. Rewrite the circunference's equation as

 $y^2 = 8 - x^2$, the upper and the lower borders of the admissible region are respectively $y = +\sqrt{8 - x^2}$ and $y = -\sqrt{8 - x^2}$. $y = +\sqrt{8-x^2}$ and $y = -\sqrt{8-x^2}$.
In the upper border consider the function $f(x, +\sqrt{8-x^2}) =$ $y^2 = 8 - x^2$, the upper and the lower borders of the admissible region are respectively $y = +\sqrt{8 - x^2}$ and $y = -\sqrt{8 - x^2}$.
In the upper border consider the function $f(x, +\sqrt{8 - x^2}) = x^3 + (\sqrt{8 - x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{8 - x^2})$ $y^2 = 8 - x^2$, the upper and the lower borders of the admissible region are respectively
 $y = +\sqrt{8 - x^2}$ and $y = -\sqrt{8 - x^2}$.

In the upper border consider the function $f(x, +\sqrt{8 - x^2}) =$
 $x^3 + (\sqrt{8 - x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{$ region are respectively
 $\frac{-2x}{\sqrt{8-x^2}} =$ $=$
 $\frac{-2x}{2\sqrt{8-x^2}}$ =
0 or 2 < x < 2 $\sqrt{2}$. Along $= +\sqrt{8-x^2}$ and $y = -\sqrt{8-x^2}$.

the upper border consider the function $f(x, +\sqrt{8-x^2}) =$
 $x^3 + (\sqrt{8-x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{8-x^2})^2 \cdot \frac{-2x}{2\sqrt{8-x^2}}$ $\frac{1}{r}$ $8 - x^2$, the upper and the lower borders of the admissible region are respectified $x + \sqrt{8 - x^2}$ and $y = -\sqrt{8 - x^2}$.

upper border consider the function $f(x, +\sqrt{8 - x^2}) =$
 $(\sqrt{8 - x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{8 - x^2})^2 \cdot \frac{-2x}{2\$ $y = +\sqrt{8-x^2}$ and $y = -\sqrt{8-x^2}$.

In the upper border consider the function $f(x, +\sqrt{8-x^2}) =$
 $x^3 + (\sqrt{8-x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{8-x^2})^2 \cdot \frac{-2x}{2\sqrt{8-x^2}} =$
 $3x(x - \sqrt{8-x^2})$; $g'(x) > 0$ if and only if $-2\sqrt{2} < x < 0$ or $2 < x <$ $x^3 + (\sqrt{8-x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{8-x^2})^2 \cdot \frac{-2x}{2\sqrt{8-x^2}} =$
 $3x(x-\sqrt{8-x^2})$; $g'(x) > 0$ if and only if $-2\sqrt{2} < x < 0$ or $2 < x < 2\sqrt{2}$. Along

the upper border, function f is increasing for $-2\sqrt{2} < x < 0$ and $2 < x < 2\sqrt{2}$ $x^3 + (\sqrt{8-x^2})^3 = g(x), g'(x) = 3x^2 + 3(\sqrt{8-x^2})^3 \cdot \frac{-2x}{2\sqrt{8-x^2}} =$
 $3x(x - \sqrt{8-x^2})$; $g'(x) > 0$ if and only if $-2\sqrt{2} < x < 0$ or $2 < x < 2\sqrt{2}$. Along

the upper border, function f is increasing for $-2\sqrt{2} < x < 0$ and $2 < x < 2\sqrt{2$ $\left(\sqrt{9-x^2}\right)^3$ ($\left(\sqrt{9-x^2}\right)^3$) ($\left(\sqrt{9-x^2}\right)^2$) ($\left(\sqrt{9-x^2}\right)^2$) ($\left(\sqrt{9-x^2}\right)^2$) ($\left(\sqrt{9-x^2}\right)^3$) ($\left(\sqrt{9-x^2}\right)^2$) ($\left(\sqrt{$

can be achaived in the lower border.

In the graphic below, the admissible region, in red, and the behaviour of f along the border rappresented by the turquoise arrows.

II M 3) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 \\ \text{u.c.: } x - y = 4 \end{cases}.$ P4
 $f(x,y) = x^2 + y^2$
 $y = 4$

a is **P4**
 $\begin{cases}\n\text{min } f(x, y) = x^2 + y^2 \\
x - y = 4\n\end{cases}$

oblem is The Lagrangian function of the problem is
 $\mathcal{L}(x, y, \lambda) = x^2 + y^2 - \lambda(x - y - 4)$ with II M 3) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x^2 + y^2 \\ \text{u.c.: } x - y = 4 \end{cases}$
The Lagrangian function of the problem is
 $\mathcal{L}(x, y, \lambda) = x^2 + y^2 - \lambda(x - y - 4)$ with
 $\nabla \mathcal{L} = (2x - \lambda, 2y + \lambda, -(x - y - 4)).$
FOC: II M 3) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x^2 + y^2 \\ \text{u.c.: } x - y = 4 \end{cases}$.
The Lagrangian function of the problem is
 $\mathcal{L}(x, y, \lambda) = x^2 + y^2 - \lambda(x - y - 4)$ with
 $\nabla \mathcal{L} = (2x - \lambda, 2y + \lambda, -(x - y - 4)).$
FOC:
 $\begin{cases} 2x - \lambda = 0 \\ 2x + \lambda = 0 \end$ The Lagrangian function of the problem is
 $\mathcal{L}(x, y, \lambda) = x^2 + y^2 - \lambda(x - y - 4)$ with
 $\nabla \mathcal{L} = (2x - \lambda, 2y + \lambda, -(x - y - 4)).$

FOC:
 $\begin{cases} 2x - \lambda = 0 \\ 2y + \lambda = 0 \end{cases}$ $\Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -\lambda/2 \end{cases}$ $\Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -2$ $\mathcal{L}(x, y, \lambda) = x^2 + y^2 - \lambda(x - y - 4)$ with
 $\nabla \mathcal{L} = (2x - \lambda, 2y + \lambda, -(x - y - 4)).$

FOC:
 $\begin{cases} 2x - \lambda = 0 \\ 2y + \lambda = 0 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -\lambda/2 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -2 \end{cases};$ one
 $\begin{cases} x = 2 \\ x - y = 4 \\ x - 4 \end{cases}$ constrai $\nabla \mathcal{L} = (2x - \lambda, 2y + \lambda, -(x - y - 4)).$

FOC:
 $\begin{cases} 2x - \lambda = 0 \\ 2y + \lambda = 0 \end{cases}$ $\Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ x - y = 4 \end{cases}$ $\Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda = 4 \end{cases}$ $\Rightarrow \begin{cases} x = 2 \\ y = -2 \text{; one} \\ \lambda = 4 \end{cases}$

constraint critical points $P = (2, -2)$.
 e Lagrangian function of the problem is
 x, y, λ = $x^2 + y^2 - \lambda(x - y - 4)$ with
 $C = (2x - \lambda, 2y + \lambda, -(x - y - 4))$.
 $2x - \lambda = 0$
 $2y + \lambda = 0 \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -\lambda/2 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -\lambda/2 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \text{; one} \\ y =$ x, y, λ) = $x^2 + y^2 - \lambda(x - y - 4)$ with
 $\mathcal{L} = (2x - \lambda, 2y + \lambda, -(x - y - 4))$.
 \mathcal{L}
 $2x - \lambda = 0$
 $2y + \lambda = 0 \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ x - y = 4 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda = 4 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \text{; one} \\ \lambda = 4 \end{cases}$ x, y, λ = $x + y - \lambda(x - y - 4)$ with
 $C = (2x - \lambda, 2y + \lambda, -(x - y - 4))$.
 $2x - \lambda = 0$
 $2y + \lambda = 0 \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ x - y = 4 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda = 4 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \text{; one} \\ \lambda = 4 \end{cases}$

astraint critical points $P = (2, -2)$. cangian function of the problem is
 λ) = $x^2 + y^2 - \lambda(x - y - 4)$ with
 $2x - \lambda, 2y + \lambda, -(x - y - 4))$.
 $\lambda = 0$ $\Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda/2 + \lambda/2 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ y = -\lambda/2 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -\lambda/2 \\ y = -\lambda/2 \end{cases}$ λ) = $x^2 + y^2 - \lambda(x - y - 4)$ with
 $2x - \lambda$, $2y + \lambda$, $-(x - y - 4)$).
 $\lambda = 0 \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ x = 4 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ x = 4 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \\ \lambda = 4 \end{cases}$ λ , $-(x - y - 4)$.
 $x = \lambda/2$
 $y = -\lambda/2$ \Rightarrow $\begin{cases} x = \lambda/2 \\ y = -\lambda/2 \end{cases}$ \Rightarrow $\begin{cases} x = 2 \\ y = -2 \text{; one} \\ \lambda = 4 \end{cases}$
 $x = \lambda/2$ \Rightarrow $\begin{cases} x = 2 \\ y = -2 \text{; one} \\ \lambda = 4 \end{cases}$ tancion of the problem is
 $+y^2 - \lambda(x - y - 4)$ with
 $2y + \lambda$, $-(x - y - 4)$).
 $\Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda/2 + \lambda/2 = 4 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda = 4 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \text{; one} \\ \lambda = 4 \end{cases}$ FOC:
 $\begin{cases} 2x - \lambda = 0 \\ 2y + \lambda = 0 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda/2 + \lambda/2 = 4 \end{cases} \Rightarrow \begin{cases} x = \lambda/2 \\ y = -\lambda/2 \\ \lambda = 4 \end{cases}$

constraint critical points $P = (2, -2)$.

SOC:
 $\begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$ $\begin{cases}\n\begin{bmatrix}\n\frac{2}{3} & \frac{1}{3} \\
x-y=4 \\
\frac{1}{3}\n\end{bmatrix}\n\end{cases}$ $\begin{cases}\n\begin{bmatrix}\n\frac{3}{3} & \frac{1}{3} \\
\frac{1}{3} & 2\n\end{bmatrix}\n\end{cases}$ $\begin{cases}\n\frac{3}{3} & \frac{1}{3} \\
\frac{1}{3} & 2\n\end{cases}$ $\begin{cases}\n\frac{3}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3}\n\end{cases}$ $\begin{cases}\n\frac{3}{3} & \frac{1}{3}$ $\lambda = 0 \Rightarrow \begin{cases} y = -\lambda/2 \\ y = -\lambda/2 \end{cases} \Rightarrow \begin{cases} z = -\lambda/2 \\ y = -\lambda/2 \end{cases} \Rightarrow \begin{cases} z = 0 \\ y = -\lambda/2 \\ \lambda = 4 \end{cases}$
 $\lambda = 4$
 $\$ y = 4 $\lambda/2 + \lambda/2 = 4$ $\lambda = 4$ $\lambda = 4$

nt critical points $P = (2, -2)$.
 $\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, with $|\overline{H}| = \begin{vmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} +$ t critical points $P = (2, -2)$.
 $0 = -1 \quad 1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 2$, with $|\overline{H}| = \begin{vmatrix} 0 & -1 & 1 \ -1 & 2 & 0 \ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \ 1 & 2 \end{vmatrix} +$
 $= -4. |\overline{H}(P)| < 0, P$ point of minimum with $f(P) = 8$. \Rightarrow $\begin{cases} y = -2; \text{ one} \\ \lambda = 4 \end{cases}$
-1 0 $\begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} =$ $\begin{vmatrix} \lambda = 4 \\ 1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} =$
 $P = 8.$ $y = -\lambda/2 \implies y = -\lambda/2$
 $-y = 4$ $\lambda/2 + \lambda/2 = 4$
 λ raint critical points $P = (2, -2)$.
 $\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, with $|\overline{H}| = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ - y = 4 $\lambda/2 + \lambda/2 = 4$ $\lambda = 4$

raint critical points $P = (2, -2)$.
 $\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, with $|\overline{\mathcal{H}}| = \begin{vmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \\ -2 = -4$. $|\overline{\mathcal{H}}(P)| < 0$, P point of minimum with $\begin{cases}\n\Rightarrow \begin{cases}\nx - \lambda/2 \\
y = -\lambda/2 \\
\lambda = 4\n\end{cases} \\
\Rightarrow \begin{cases}\nx - 2 \\
y = -2 \\
\lambda = 4\n\end{cases} \\
\Rightarrow \begin{cases}\n0 & -1 & 1 \\
-1 & 2 & 0 \\
1 & 2 & 0\n\end{cases} = \begin{vmatrix} -1 & 0 \\
1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 0 \\
-1 & 2 & 0 \\
1 & 2 & 0\n\end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\
-1 & 2 & 0 \\
1 & 2 & 0\n\end{vmatrix} + \begin{vmatrix} -1 & 0 & 0$ $/2 + \lambda/2 = 4$ $\lambda = 4$ $\lambda = 4$ $\lambda = 4$
 $P = (2, -2)$.

, with $|\overline{H}| = \begin{vmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} =$
 $| < 0, P$ point of minimum with $f(P) = 8$. SOC:
 $\overline{H} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, with $|\overline{H}| = \begin{vmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} = -2 - 2 = -4$. $|\overline{H}(P)| < 0$, P point of minimum with $f(P) = 8$.

II M $\overline{\mathcal{H}} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, with $|\overline{\mathcal{H}}| = \begin{vmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} =$
 $-2 - 2 = -4$. $|\overline{\mathcal{H}}(P)| < 0$, P point of minimum with $f(P) = 8$ h $|\overline{\mathcal{H}}| = \begin{vmatrix} -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} =$

0, *P* point of minimum with $f(P) = 8$.
 $(x, y) = e^{x+y} - e^{x-y}$ and the unit vector $v = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$;

directional derivat $\begin{aligned}\n\begin{aligned}\n&\begin{bmatrix}\n1 & 0 & 2 \\
-2 - 2 &= -4.\n\end{bmatrix} \n\end{aligned} \n\left| \frac{7}{7}(P) \right| < 0, P \text{ point of minimum with } f(P) = 8.\n\end{aligned}\n\end{aligned}$ II M 4) Given the function $f(x, y) = e^{x+y} - e^{x-y}$ and the unit vector $v = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right);$ calculate on p II M 4) Given the function $f(x, y) = e^{x+y} - e^{x-y}$ and the unit vector $v = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$;

calculate on point (0, 0) the directional derivatives $\mathcal{D}_{v}f(0,0)$ and $\mathcal{D}_{v,v}^{(2)}f(0,0)$.

$$
\nabla f(x, y) = (e^{x+y} - e^{x-y}, e^{x+y} + e^{x-y}), \nabla f(0, 0) = (0, 2),
$$

\n
$$
\mathcal{D}_v f(0, 0) = \nabla f(0, 0) \cdot v = (0, 2) \cdot \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \sqrt{3}.
$$

\n
$$
\mathcal{H}f(x, y) = \begin{bmatrix} e^{x+y} - e^{x-y} & e^{x+y} + e^{x-y} \\ e^{x+y} + e^{x-y} & e^{x+y} - e^{x-y} \end{bmatrix} \text{ and } \mathcal{H}f(0, 0) = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \text{ with}
$$

\n
$$
\mathcal{D}_{v,v}^{(2)} f(0, 0) = v^T \cdot \mathcal{H}f(0, 0) \cdot v = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \cdot \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \cdot \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) =
$$

\n
$$
\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \cdot \left(\sqrt{3}\right) = \sqrt{3}.
$$