## **UNIVERSITA' DEGLI STUDI DI SIENA Scuola di Economia e Management A.A. 2023/24 Quantitative Methods for Economic Applications - Mathematics for Economic Applications Task 3/6/2024 IM 1)** Given the complex number  $z = \frac{(1-i)^2}{(1+i)^2}$ , calculate its square roots.<br>  $(1-i)^2$   $1-2i+i^2$   $1-2i-1$   $-2i$ Economic Applica<br>
Economic Applica<br>  $(\frac{3}{6}/2024)$ <br>  $\frac{1-i}{(1+i)^2}$ , calculate its squ **Economic Applicat**<br> **k 3/6/2024**<br>  $\frac{(1-i)^2}{(1+i)^2}$ , calculate its squa

**ECONOMIC Applica**<br>  $\frac{3}{6/2024}$ <br>  $\frac{1-i}{1+i}$ , calculate its squ<br>  $\frac{1}{1+i} = \frac{-2i}{2i} = -1 = co$ **k** 3/6/2024<br>  $\frac{(1-i)^2}{(1+i)^2}$ , calculate its squa<br>  $\frac{-1}{-1} = \frac{-2i}{2i} = -1 = \cos \theta$  $\overline{2}$  $_2$ , calc

I M 1) Given the complex number  $z = \frac{(1-i)^2}{(1+i)^2}$ , calculate its square roots.<br>  $z = \frac{(1-i)^2}{(1+i)^2} = \frac{1-2i+i^2}{1+2i+i^2} = \frac{1-2i-1}{1+2i-1} = \frac{-2i}{2i} = -1 = \cos \pi + i \sin \pi$ . For the square roots we apply the classical formula: **Task 3/6/2024**<br>
Given the complex number  $z = \frac{(1-i)^2}{(1+i)^2}$ , calculate its square re<br>  $\frac{1-i}{(1+i)^2} = \frac{1-2i+i^2}{1+2i+i^2} = \frac{1-2i-1}{1+2i-1} = \frac{-2i}{2i} = -1 = \cos \pi + i$ Given the complex number  $z = \frac{(1 - i)^2}{(1 + i)^2}$ , calculate its square<br>  $\frac{1 - i)^2}{(1 + i)^2} = \frac{1 - 2i + i^2}{1 + 2i + i^2} = \frac{1 - 2i - 1}{1 + 2i - 1} = \frac{-2i}{2i} = -1 = \cos \pi$ <br>
croots we apply the classical formula: ) Given the complex numb<br>  $\frac{(1-i)^2}{(1+i)^2} = \frac{1-2i+i^2}{1+2i+i^2} =$ <br>
e roots we apply the classic ) Given the complex numb<br>  $\frac{(1-i)^2}{(1+i)^2} = \frac{1-2i+i^2}{1+2i+i^2} =$ <br>
e roots we apply the classic<br>  $= \sqrt{\cos \pi + i \sin \pi} = \cos \theta$ 2  $\overline{1}$  $2^{\sim}1$ . 2  $1$ . the state of  $\frac{(1-i)^2}{(1+i)^2}$ , calculate its square roots.<br>  $\frac{2}{2} = \frac{1-2i-1}{1+2i-1} = \frac{-2i}{2i} = -1 = \cos \pi + i \sin \pi$ . For the assical formula: square roots we apply the classical formula:  $z = \frac{(1-i)^2}{(1+i)^2} = \frac{1-2i+i^2}{1+2i+i^2} = \frac{1-2i-1}{1+2i-1} = \frac{-2i}{2i} = -1 = \cos \pi + i \sin \pi$ . For the<br>square roots we apply the classical formula:<br> $\sqrt{z} = \sqrt{\cos \pi + i \sin \pi} = \cos \left(\frac{\pi}{2} + k\pi\right) + i \sin \left(\frac{\pi}{2} + k\pi\right) k = 0, 1$ . The two roots<br>are  $(1+i)^2$ <br>=  $\frac{1-2i+i^2}{1+2i+i^2} = \frac{1-2i-1}{1+2i-1} = \frac{-2i}{2i} = -1 = \cos \pi + i \sin \pi$ .<br>we apply the classical formula:<br> $\pi + i \sin \pi = \cos \left(\frac{\pi}{2} + k\pi\right) + i \sin \left(\frac{\pi}{2} + k\pi\right)$   $k = 0, 1$ . The two  $\text{Tr } z = \frac{z}{(1+i)^2}$ , calculate its square roots.<br>  $\frac{1-2i-1}{1+2i-1} = \frac{-2i}{2i} = -1 = \cos \pi + i \sin \pi$ . For the all formula:<br>  $\frac{\pi}{2} + k\pi + i \sin \left(\frac{\pi}{2} + k\pi\right) k = 0, 1$ . The two roots

$$
\sqrt{z} = \sqrt{\cos \pi + i \sin \pi} = \cos \left(\frac{\pi}{2} + k\pi\right) + i \sin \left(\frac{\pi}{2} + k\pi\right) \ k = 0, 1.
$$
 The two roots

are:

$$
z = \frac{z - \overline{(1 + i)^2}}{1 + 2i + i^2} - \frac{1}{1 + 2i - 1} - \frac{1}{2i} - \frac{1}{2i} - \frac{1}{1 + 2i - 1}
$$
  
\nsquare roots we apply the classical formula:  
\n
$$
\sqrt{z} = \sqrt{\cos \pi + i \sin \pi} = \cos \left(\frac{\pi}{2} + k\pi\right) + i \sin \left(\frac{\pi}{2} + k\pi\right) k = 0
$$
  
\nare:  
\n
$$
k = 0 \rightarrow z_0 = \cos \left(\frac{\pi}{2}\right) + i \sin \left(\frac{\pi}{2}\right) = i
$$
  
\n
$$
k = 1 \rightarrow z_1 = \cos \left(\frac{3\pi}{2}\right) + i \sin \left(\frac{3\pi}{2}\right) = -i
$$
  
\nIM 2) Given the matrix  $\mathbb{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ , calculate its eigenvalues

 $\lim_{t \to 0} \left( \frac{3\pi}{2} \right) = -i.$ <br>  $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ , calculate its eigenvalues of the subset of the su  $\begin{pmatrix} 2\sin\left(\frac{3\pi}{2}\right) = -i. \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix}$ , calculate its eigenvalues and study if the<br>not.<br>he characteristic polynomial of matrix  $\mathbb{A}$ .  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ , calculate its eigenval<br>
e or not.<br>
late the characteristic polynomial of ma<br>  $\begin{bmatrix} -1 & 0 & -2 \\ 0 & \lambda - 1 & 0 \\ 4 & 0 & \lambda - 1 \end{bmatrix} = (\lambda - 1) \begin{bmatrix} \lambda - 1 \\ -4 \end{bmatrix}$ ix  $\mathbb{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ , calculate its eigenvalue<br>
zable or not.<br>
lculate the characteristic polynomial of matri<br>  $\begin{vmatrix} \lambda - 1 & 0 & -2 \\ 0 & \lambda - 1 & 0 \\ 4 & 0 & \lambda - 1 \end{vmatrix} = (\lambda - 1) \begin{vmatrix} \lambda - 1 & 0 \\ -4 & 0 \end{vmatrix$ 

matrix  $A$  is diagonalizable or not.

At the first step we calculate the characteristic polynomial of matrix  $\mathbb{A}$ ;

I M 2) Given the matrix 
$$
A = \begin{bmatrix} 0 & 1 & 0 \ 4 & 0 & 1 \end{bmatrix}
$$
, calculate its eigenvalues and study if the  
matrix A is diagonalizable or not.  
At the first step we calculate the characteristic polynomial of matrix A;  

$$
P_{\mathbb{A}}(\lambda) = |\lambda \mathbb{I} - \mathbb{A}| = \begin{vmatrix} \lambda - 1 & 0 & -2 \ 0 & \lambda - 1 & 0 \ -4 & 0 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} \lambda - 1 & -2 \ -4 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} \lambda - 1 & -2 \ -4 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} \lambda - 1 & -2 \ \lambda - 1 & -2 \end{vmatrix}
$$
 $(\lambda - 1)((\lambda - 1)^2 - 8)$ . Putting  $P_{\mathbb{A}}(\lambda) = 0$  we find the three eigenvalues of matrix A; if

At the first step we calculate the characteristic polynomial of matrix A;<br>  $P_{\mathbb{A}}(\lambda) = |\lambda \mathbb{I} - \mathbb{A}| = \begin{vmatrix} \lambda - 1 & 0 & -2 \\ 0 & \lambda - 1 & 0 \\ -4 & 0 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 1 \end{vmatrix} = (\lambda - 1)((\lambda - 1)^2 - 8)$ . Putting  $P_{\mathbb$  $(\lambda - 1)((\lambda - 1)^2 - 8)$ . Putting  $P_{\mathbb{A}}(\lambda) = 0$  we find the three eigenvalues of matrix  $\mathbb{A}$ ; if  $P_{\mathbb{A}}(\lambda) = |\lambda \mathbb{I} - \mathbb{A}| = \begin{vmatrix} \lambda - 1 & 0 & -2 \\ 0 & \lambda - 1 & 0 \\ -4 & 0 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} (\lambda - 1)((\lambda - 1)^2 - 8) \end{vmatrix}$ . Putting  $P_{\mathbb{A}}(\lambda) = 0$  we find the three eigenvalues of matrix A; if  $P_{\mathbb{A}}(\lambda) = |\lambda \mathbb{I} - \mathbb{A}| = \begin{vmatrix} 0 & \lambda - 1 & 0 \\ -4 & 0 & \lambda - 1 \end{vmatrix} = (\lambda - 1)\begin{vmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 1 \end{vmatrix} = (\lambda - 1)((\lambda - 1)^2 - 8)$ . Putting  $P_{\mathbb{A}}(\lambda) = 0$  we find the three eigenvalues of n  $\lambda - 1 = 0$ , we have the first eigenvalue  $(\lambda - 1)^2 = 8$  and  $\lambda - 1 = \pm 2\sqrt{2}$ , thus  $\lambda_{2,3} = 1 \pm 2\sqrt{2}$ . The three eigenvalues are one to one differents, thus matrix  $A$  is a diagonalizable one.  $(\lambda - 1)((\lambda - 1)^2 - 8)$ . Putting  $P_A(\lambda) = 0$  we find the three eigenvalue  $\lambda - 1 = 0$ , we have the first eigenvalue  $\lambda_1 = 1$ ; if  $(\lambda - 1)^2 - 8 = 0$ <br>  $(\lambda - 1)^2 = 8$  and  $\lambda - 1 = \pm 2\sqrt{2}$ , thus  $\lambda_{2,3} = 1 \pm 2\sqrt{2}$ . The three one to

 $\lambda - 1 = 0$ , we have the first eigenvalue  $\lambda_1 = 1$ ; if  $(\lambda - 1)^2 - 8 = 0$  it follow  $(\lambda - 1)^2 = 8$  and  $\lambda - 1 = \pm 2\sqrt{2}$ , thus  $\lambda_{2,3} = 1 \pm 2\sqrt{2}$ . The three eigenvalues are one to one differents, thus matrix  $\mathbb A$  is a dia Find the matrix associated with the linear application, calculate the dimentions of both, kernel and immage of  $F$ , and find a basis for the kernel and a basis for the image. I M 3) Given the linear application  $F: \mathbb{R}^3 \to \mathbb{R}^2$ , we know that:<br>1.  $F(1, 0, 0) = (1, 0);$  <br>2.  $F(1, 1, 0) = (1, 1);$  <br>3. I the matrix  $P: \mathbb{R} \to \mathbb{R}$ , we know that.<br>
(0); 2.  $F(1, 1, 0) = (1, 1);$  3.  $F(1, 1, 1) = (1, 1).$ <br>
ociated with the linear application, calculate the dimentions of both,<br>
of F, and find a basis for the kernel and a basis f

1. 
$$
P(1, 0, 0) = (1, 0),
$$
  
\n2.  $P(1, 1, 0) = (1, 1),$   
\n3.  $P(1, 1, 1) = (1, 1).$   
\nFind the matrix associated with the linear application, calculate the dimensions of  
\nkernel and immage of *F*, and find a basis for the kernel and a basis for the image.  
\nDefine with  $A_F = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$  the matrix associated to the linear application, by  
\nconditions 1., 2. and 3. we know  $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} =$   
\n $\begin{bmatrix} a & a+b & a+b+c \\ d & d+e & d+e+f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ , thus  $a = 1, b = 0, c = 0, d = 0, e = 1$ 

Find the matrix associated with the linear application, calculate the dimentions of<br>kernel and immage of F, and find a basis for the kernel and a basis for the image<br>Define with  $A_F = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$  the matrix ass

Define with 
$$
\mathbb{A}_F = \begin{bmatrix} a & b \\ d & e & f \end{bmatrix}
$$
 the matrix associated to the linear application, by  
conditions 1., 2. and 3. we know  $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} =$   
 $\begin{bmatrix} a & a+b & a+b+c \\ d & d+e & d+e+f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ , thus  $a = 1, b = 0, c = 0, d = 0, e = 1$  and  
 $f = 0$ ;  $\mathbb{A}_F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ . Matrix  $\mathbb{A}_F$  has rank equal 2 and the dimension of the image  
is 2 while the dimension of the kernel is 1; for the basis of the image we can note that  
the codomain of F is the set  $\mathbb{R}^2$  and the dimension of the image is 2 thus  $\text{Im}a(F) = \mathbb{R}^2$ 

and a base for it is the set  $\mathcal{B}_{Ima(F)} = \{(1,0), (0,1)\}$ . For a basis of the kernel we take a generic element of the domain  $(x, y, z)$ , its image is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ x \\ y \end{pm$ and a base for it is the set  $\mathcal{B}_{Ima(F)} = \{(1,0), (0,1)\}\)$ . For a basis of the kernel we take a<br>generic element of the domain  $(x, y, z)$ , its image is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ , thus<br> $(x$  $y = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$  $z \leftarrow \sqrt{z}$  $x\Big\}$  there  $y \int$ , thus (1, 0), (0, 1)}. For a basis of the kernel we take a<br>
, its image is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ , thus<br>
aly if  $x = y = 0$  and we conclude that a generic

and a base for it is the set  $B_{Ima(F)} = \{(1,0), (0,1)\}\)$ . For a basis of the kernel we take a generic element of the domain  $(x, y, z)$ , its image is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ , thus  $(x, y, z$ generic element of the domain  $(x, y, z)$ , its image is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ <br>  $(x, y, z)$  belongs on the kernel if and only if  $x = y = 0$  and we conclude that a ge<br>
element of the ker

generic element of the domain  $(x, y, z)$ , its image is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ , thus  $(x, y, z)$  belongs on the kernel if and only if  $x = y = 0$  and we conclude that a generic element o  $(x, y, z)$  belongs on the kernel if and only if  $x = y = 0$  and we conclude that a generic<br>element of the kernel is  $(0, 0, z) = z(0, 0, 1)$ . A basis for the kernel is the set<br> $\mathcal{B}_{Ker(F)} = \{(0, 0, 1)\}$ .<br>I M 4) Vector V has coordin (*x*, *y*, *z*) detongs on the kerner in an<br>element of the kernel is  $(0, 0, z) =$ <br> $\mathcal{B}_{Ker(F)} = \{(0, 0, 1)\}.$ <br>I M 4) Vector *V* has coordinates (1<br>coordinates  $(-1, -1, -1)$  respector  $(x, y, z)$ .<br>If vector *V* has coordinates  $(1,$  $B_{Ker(F)} = \{(0,0,1)\}\$ .<br>
I M 4) Vector V has coordinates  $(1,1,1)$  respect the basis  $B = \{(0,0,1), (0,1)\}$ .<br>
coordinates  $(-1,-1,-1)$  respect the basis  $B' = \{(1,0,0), (1,1,0), (x, y, z)\}$ <br>
vector  $(x, y, z)$ .<br>
If vector V has coordinates  $(1$ 

I M 4) Vector *V* has coordinates  $(1, 1, 1)$  respect the basis  $B = \{(0, 0, 1), (0, 1, 1), (1, 1, 1)\}$ , and<br>coordinates  $(-1, -1, -1)$  respect the basis  $B' = \{(1, 0, 0), (1, 1, 0), (x, y, z)\}$ . Determine the<br>vector  $(x, y, z)$ .<br>If vector TM 4) Vector V has coordinates  $(1,1,1)$  respect the basis  $B' = \{(0,0,1), (0,1,1), (1,1,1)\}$ , and<br>coordinates  $(-1, -1, -1)$  respect the basis  $B' = \{(1,0,0), (1,1,0), (x, y, z)\}$ . Determine the<br>vector  $(x, y, z)$ .<br>If vector V has coordinat coordinates  $(-1, -1, -1)$  respect the basis  $B = \{(1, 0, 0), (1, 1, 0), (x, y, z)\}$ . Determinive<br>vector  $(x, y, z)$ .<br>If vector V has coordinates  $(1, 1, 1)$  respect the basis B and coordinates<br> $(-1, -1, -1)$  respect the basis  $B', V = 1 \cdot (0$ If vector  $(x, y, z)$ .<br>
If vector  $V$  has coordinates  $(1, 1, 1)$  respect the basis  $B$  and coordinates<br>  $(-1, -1, -1)$  respect the basis  $B', V = 1 \cdot (0, 0, 1) + 1 \cdot (0, 1, 1) + 1 \cdot (1, 1, 1) =$ <br>  $(1, 2, 3)$  and at the same time  $V = -1 \$  $= 1 \cdot (0, 0, 1) + 1 \cdot (0, 1, 1) + 1 \cdot (1, 1, 0) - 1 \cdot (x, y, z)$ <br>  $= (-2 - x, -1 - y, -z)$  it easily<br>  $e^{x+y} - e^{y+z} = 0$  satisfied at the po<br>  $x - z + xyz = 0$  $(1,0,0)-1\cdot(1,1,0)-1\cdot(x,y,z)$ <br>  $e^{x+y}-e^{y+z}=0$ <br>  $x-z+xyz=0$  satisfied at the point

(1, 2, 3) and at the same time  $v = -1$  (1, 0, 0)  $-1$  (1, 1, 0)  $-1$  ( $x, y, z$ )  $-$ <br>  $(-2-x, -1-y, -z)$ . Put  $(1, 2, 3) = (-2-x, -1-y, -z)$  it easily follows<br>  $x = y = z = -3$ .<br>
II M 1) Given the system of equations  $\begin{cases} e^{x+y} - e^{y+z} = 0 \\ x - z +$  $x = y = z = -3.$ <br>
II M 1) Given the system of equations  $\begin{cases} e^{x+y} - e^{y+z} = 0 \\ x - z + xyz = 0 \end{cases}$  satisfied at the point  $P(1,0,1)$ ; verify that with it an implicit function  $z \mapsto (x(z), y(z))$  can be defined and then calculate, for this If M 1) Given the system of equations  $\begin{cases} e^{x+y}-e^{y+z}=0 \\ x-z+xyz=0 \end{cases}$  satisfied at the point  $P(1,0,1)$ ; verify that with it an implicit function  $z \mapsto (x(z), y(z))$  can be defined and then calculate, for this implicit function, II M 1) Given the system of equations  $\begin{cases} c & -b & -b \\ x - z + xyz = 0 & \text{satisfied at the point} \end{cases}$ <br>  $P(1,0,1)$ ; verify that with it an implicit function  $z \mapsto (x(z), y(z))$  can be defined and then calculate, for this implicit function, the derivativ  $P(1,0,1)$  is is a contract the function  $F: \mathbb{R}^3 \to \mathbb{R}^2$  with  $F(x, y, z) = (e^{x+y} - e^{y+z}, x - z + xyz)$  and<br>
e jacobian matrix  $\mathbb{J} = \begin{bmatrix} e^{x+y} & e^{x+y} - e^{y+z} & -e^{y+z} & -z + xyz \ 1 + yz & xz & -1 + xy \end{bmatrix}$ . Matrix  $\mathbb{J}$  at point<br>
(1, 0, 1) is  $\begin{bmatrix} e &$ the jacobian matrix  $\mathbb{J} = \begin{bmatrix} e^{x+y} & e^{x+y} - e^{y+z} & -e^{y+z} \\ 1+yz & xz & -1+xy \end{bmatrix}$ . Matrix  $\mathbb{J}$  at point<br>  $P(1,0,1)$  is  $\begin{bmatrix} e & 0 & -e \\ 1 & 1 & -1 \end{bmatrix}$  and the minor of its restriction respect variables x and y is<br>  $|\mathbb{J}($  $P(1,0,1)$  is  $\begin{bmatrix} e & 0 & -e \\ 1 & 1 & -1 \end{bmatrix}$  and the minor of its restriction respect variables x and y is<br>  $|\mathbb{J}(P)|_{x,y}| = \begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix} = e \neq 0$ , the proposed system of equations define an implicit<br>
function  $z \mapsto$  $|\mathbb{J}(P)|_{x,y}| = \begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix} = e \neq 0$ , the proposed system of equations define<br>function  $z \mapsto (x(z), y(z))$  on a neighborhood of the point P. For the de<br>have  $x'(1) = -\frac{\begin{vmatrix} -e & 0 \\ -1 & 1 \end{vmatrix}}{\begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix}} = 1$  and - 1 and the minor of its restriction respect variables x a<br>
=  $e \neq 0$ , the proposed system of equations define an imp<br>  $y(z)$  on a neighborhood of the point P. For the derivative<br>  $\begin{vmatrix} e & -e \\ 1 & 1 \end{vmatrix} = 1$  and  $y'(1) = -\frac$  $e = e \neq 0$ , the proposed system of equations define an imp<br>  $y(z)$  on a neighborhood of the point *P*. For the derivative<br>  $\begin{vmatrix} e & 0 \\ 1 & 1 \\ 0 & 0 \end{vmatrix} = 1$  and  $y'(1) = -\frac{1}{e} \begin{vmatrix} e & -e \\ 1 & -1 \\ e & 0 \end{vmatrix} = 0$ .  $e = e \neq 0$ , the proposed system of equations define an in<br>  $e, y(z)$  on a neighborhood of the point *P*. For the derivativ<br>  $\begin{vmatrix} -e & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$  and  $y'(1) = -\frac{e^{z} - e^{z}}{e^{z} - e^{z}} = 0$ .  $\begin{vmatrix} 1 & y(z) & \text{on a neighborhood of the point } P. \text{ For the derivative } 0 - e & 0 \\ \frac{-1 & 1}{e & 0} & = 1 \text{ and } y'(1) = -\frac{\begin{vmatrix} e & -e \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix}} = 0.$ <br>
bblem  $\begin{cases} \text{Max/min } f(x, y) = x \cdot y \\ 0 & \text{otherwise} \end{cases}$  $\begin{aligned}\n\mathcal{L}_{x,y}|&= \begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix} = e \neq 0, \text{ the proposed system of equations define an implicit}\n\mathbf{n} \quad z \mapsto (x(z), y(z)) \text{ on a neighborhood of the point } P. \text{ For the derivatives we} \\
\begin{vmatrix} -e & 0 \\ -1 & 1 \end{vmatrix} &= 1 \text{ and } y'(1) = -\frac{\begin{vmatrix} e & -e \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix}} = 0.\n\end{aligned}$  $\begin{vmatrix} 0 \\ 1 \\ z \end{vmatrix} = e \neq 0$ , the proposed system of equations define an implies  $z$ ,  $y(z)$  on a neighborhood of the point *P*. For the derivatives  $\begin{vmatrix} -e & 0 \\ -1 & 1 \\ e & 0 \end{vmatrix} = 1$  and  $y'(1) = -\frac{\begin{vmatrix} e & -e \\ 1 & -1 \\ e & 0 \end{vmatrix$  $(y, y(z))$  on a neighborhood of the point *P*. For the derivative<br>  $\begin{array}{c|c} -e & 0 \\ -1 & 1 \end{array}$ <br>  $\begin{array}{|c|c|c|c|c|c|c|c|} \hline e & 0 & e & e \\ 1 & -1 & -1 & e \\ 1 & 1 & 1 & 1 \end{array}$ <br>  $\begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 \end{array}$ <br>  $\hline$  $y'(1) = -\frac{\begin{vmatrix} e & -e \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} e & 0 \\ 1 & 1 \end{vmatrix}} = 0.$ <br>  $f(x, y) = x \cdot y$ <br>  $y^2 \le 4$ <br>
to the admissible region and  $y'(1) = -\frac{|1 - 1|}{|e| \cdot 0|} = 0$ <br>  $\left| \begin{array}{cc} \tan f(x, y) = x \cdot y \\ 1 & 1 \end{array} \right|$ <br>  $x^2 + y^2 \le 4$ <br>
ontinuos function, the admissible

II M 2) Solve the problem  $\begin{cases} \text{Max/min } f(x, y) = x \cdot y \\ u.c. : x^2 + y^2 \leq 4 \end{cases}.$ 

The function  $f$  is a polynomial, continuos function, the admissible region is a disk with If  $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ <br>II M 2) Solve the problem  $\begin{cases} \text{Max/min } f(x, y) = x \cdot y \\ \text{u.c.: } x^2 + y^2 \le 4 \end{cases}$ .<br>The function f is a polynomial, continuos function, the admissible region is a disk with center (0, 0) and radius 2, maximum and minimum in the admissible region. The Lagrangian function is II M 2) Solve the problem  $\begin{cases} \text{max/min } f(x, y) = x & y \\ \text{u.c.: } x^2 + y^2 \le 4 \end{cases}$ .<br>The function f is a polynomial, continuos function, the a center (0, 0) and radius 2, a bounded and closed set, ther maximum and minimum in the adm

The function *f* is a polynomial, continuous function, the admissible region is a disk with center (0, 0) and radius 2, a bounded and closed set, therefore *f* presents absolute maximum and minimum in the admissible region. The Lagrangian function is\n
$$
\mathcal{L}(x, y, \lambda) = x \cdot y - \lambda(x^2 + y^2 - 4)
$$
\nwith\n
$$
\nabla \mathcal{L} = (y - 2\lambda x, x - 2\lambda y, -(x^2 + y^2 - 4)).
$$
\n
$$
\Gamma^{\circ} \mathcal{C} \mathcal{A} \mathcal{S} \mathcal{E} \text{ (free optimization):}
$$
\n
$$
\begin{cases}\n\lambda = 0 \\
y = 0 \\
x = 0\n\end{cases}\n\Rightarrow\n\begin{cases}\n\lambda = 0 \\
y = 0 \\
x = 0\n\end{cases}\n\Rightarrow\n\begin{cases}\n\lambda = 0 \\
y = 0 \\
x = 0\n\end{cases}; \mathcal{H}f = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ with determinant } |\mathcal{H}f| = -1 < 0, (0, 0) \\
x^2 + y^2 \le 4 \\
0 \le 4\n\end{cases}
$$
\nis a saddle point.\n
$$
II^{\circ} \mathcal{C} \mathcal{A} \mathcal{S} \mathcal{E} \text{ (constrained optimization):}
$$

is a saddle point.

$$
\begin{cases}\n\lambda \neq 0 & \text{if } y = 2\lambda x \\
y - 2\lambda y = 0 \Rightarrow \begin{cases}\n\lambda \neq 0 \\
x - 4\lambda^2 x = 0 \\
x^2 + y^2 \neq 4 \\
x^2 + y^2 \neq 4\n\end{cases} \text{ otherwise if } 1 - 4\lambda^2 = 0, \lambda = \pm 1/2, y = \pm x \text{ and by the condition } x^2 + y^2 = 4 \text{ otherwise if } 1 - 4\lambda^2 = 0, \lambda = \pm 1/2, y = \pm x \text{ and by the condition } x^2 + y^2 = 4 \text{ we get } x = \pm \sqrt{2}; \text{ four constrained critical points } P_1(\sqrt{2}, \sqrt{2}), P_2(-\sqrt{2}, -\sqrt{2}), P_3(\sqrt{2}, -\sqrt{2}), P_4(-\sqrt{2}, \sqrt{2}). \text{ Two of these } (\pm \sqrt{2}, \pm \sqrt{2}) \text{ have } \lambda > 0, \text{ candidate for maximum, the others } (\pm \sqrt{2}, \mp \sqrt{2}) \text{ have } \lambda > 0, \text{ candidate for maximum, the others } (\pm \sqrt{2}, \mp \sqrt{2}) \text{ have } \lambda > 0, \text{ candidate for maximum, the other } (\pm \sqrt{2}, \mp \sqrt{2}) \text{ have } \lambda > 0, \text{ remainder of the maximum, } \lambda > 0 \text{ for } \lambda > 0 \text{ if } \lambda
$$

Remember that 
$$
\mathcal{D}_{v,w}^{(2)} f(x_P, y_P) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \mathcal{H} f(x_P, y_P) \cdot \left(\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right)
$$
 and  
\n
$$
\mathcal{H} f(x_P, y_P) = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}; \text{ we get}
$$
\n
$$
\mathcal{D}_{v,w}^{(2)} f(x_P, y_P) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \cdot \left(\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right) =
$$
\n
$$
\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left(-\sqrt{2}, -\sqrt{2}\right) = -2.
$$