## **UNIVERSITA' DEGLI STUDI DI SIENA Scuola di Economia e Management A.A. 2023/24 Quantitative Methods for Economic Applications - Mathematics for Economic Applications Task 26/8/2024 conomic Application**<br>26/8/2024<br> $\frac{1}{i} - \frac{1}{1-i}$ , find the real par

**Mathematics for Economic Applications<br>
Task 26/8/2024**<br>
I M 1) Given the complex number  $z = \frac{1}{1+i} - \frac{1}{1-i}$ , find the real part and the<br>
imaginary part of z and calculate its cubic roots. ECONOMIC Applications<br>  $\frac{1}{1+i} - \frac{1}{1-i}$ , find the real part a<br>  $\frac{1}{1+i} - \frac{1}{1-i}$ , find the real part a<br>  $-2i = 2i$ imaginary part of  $z$  and calculate its cubic roots. I M 1) Given the complex number  $z = \frac{1}{1+i} - \frac{1}{1-i}$ , find the real part and the<br>maginary part of z and calculate its cubic roots.<br> $z = \frac{1}{1+i} - \frac{1}{1-i} = \frac{1-i - (1+i)}{(1+i)(1-i)} = \frac{-2i}{1-i^2} = \frac{-2i}{2} = -i$  (remember that<br> $i^2 = -1$ ). ) Given the complex number  $z = \frac{1}{1+i} - \frac{1}{1-i}$ , find the real part and inary part of z and calculate its cubic roots.<br>  $\frac{1}{1+i} - \frac{1}{1-i} = \frac{1-i - (1+i)}{(1+i)(1-i)} = \frac{-2i}{1-i^2} = \frac{-2i}{2} = -i$  (rememb - 1).  $Re(z) = 0$ ,  $Im(z) = -1$ . In g ber  $z = \frac{1}{1+i} - \frac{1}{1-i}$ , fin<br>te its cubic roots.<br> $\frac{(1+i)}{(1-i)} = \frac{-2i}{1-i^2} = \frac{-2i}{2}$ <br>= -1. In goniometric form ex number  $z = \frac{1}{1+i} - \frac{1}{1-i}$ , fi<br>
calculate its cubic roots.<br>  $\frac{1-i-(1+i)}{(1+i)(1-i)} = \frac{-2i}{1-i^2} = \frac{-2i}{2}$ <br>  $Im(z) = -1$ . In goniometric for (remember that I M 1) Given the complex number  $z = \frac{1}{1+i} - \frac{1}{1-i}$ , find the r<br>imaginary part of z and calculate its cubic roots.<br> $z = \frac{1}{1+i} - \frac{1}{1-i} = \frac{1-i - (1+i)}{(1+i)(1-i)} = \frac{-2i}{1-i^2} = \frac{-2i}{2} = -i$ <br> $i^2 = -1$ ).  $Re(z) = 0$ ,  $Im(z) = -1$ . In goniom imaginary part of z and calculate its cubic ro<br>  $z = \frac{1}{1+i} - \frac{1}{1-i} = \frac{1-i-(1+i)}{(1+i)(1-i)} = \frac{-i}{1-i}$ <br>  $z^2 = -1$ ).  $Re(z) = 0$ ,  $Im(z) = -1$ . In go<br>  $z = -i = \cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi$ .  $1 + i$ <br>  $\frac{1}{z}$  and calculate its cubic roots.<br>  $\frac{1}{z - i} = \frac{1 - i - (1 + i)}{(1 + i)(1 - i)} = \frac{-2i}{1 - i^2}$ <br>  $\frac{2}{z}$ <br>  $\frac{3}{2} \pi + i \sin \frac{3}{2} \pi$ . imaginary part of z and calculate its cubic roots.<br>  $z = \frac{1}{1+i} - \frac{1}{1-i} = \frac{1-i-(1+i)}{(1+i)(1-i)} = \frac{-2i}{1-i^2} = \frac{-2i}{2} = -i$  (remember th<br>  $i^2 = -1$ ).  $Re(z) = 0$ ,  $Im(z) = -1$ . In goniometric form:<br>  $z = -i = \cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi$ .<br>
For the  $\frac{1}{2}$  =  $-\iota$  (remember that<br>  $\tau$  form:<br>  $\tau = \sqrt[3]{\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi} =$ <br>  $\pi + \frac{2}{\ln 2}i \sin \left(\frac{\pi}{2} + \frac{2}{\ln 2}\right)$ (remember that<br>  $\frac{3}{2}\pi + i \sin \frac{3}{2}\pi =$ 

 $\frac{3}{2}\pi + i\sin\frac{3}{2}\pi =$ <br> $+ i\sin\left(\frac{\pi}{2} + \frac{2}{3}k\pi\right)$  $\pi + i \sin \frac{3}{2}\pi =$  $z^2 = -1$ .  $Re(z) = 0$ ,  $Im(z) = -1$ . In goniometric form:<br>  $z = -i = \cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi$ .<br>
For the cubic roots we apply the classical formula:  $\sqrt[3]{z} = \sqrt[3]{\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi} =$ <br>  $\cos \left(\frac{3\pi/2 + 2k\pi}{3}\right) + i \sin \left(\frac{3\pi/2 + 2k\pi$  $\frac{3}{2}\pi + i \sin \frac{3}{2}\pi =$ <br>  $\pi$  +  $i \sin \left(\frac{\pi}{2} + \frac{2}{3}k\pi\right)$ For the cubic roots we apply the clas<br>  $cos\left(\frac{3\pi/2 + 2k\pi}{3}\right) + i sin\left(\frac{3\pi/2}{k}\right)$ <br>  $k = 0, 1, 2$ . The three roots are:<br>  $k = 0 \rightarrow z_0 = cos\left(\frac{\pi}{2}\right) + i sin\left(\frac{\pi}{2}\right)$ For the cubic roots we apply the classical formula:  $\sqrt[3]{z} = \sqrt[3]{\frac{z^2}{c^2}}$ <br>  $\cos\left(\frac{3\pi/2 + 2k\pi}{3}\right) + i \sin\left(\frac{3\pi/2 + 2k\pi}{3}\right) = \cos\left(\frac{\pi}{2} + \frac{2}{3}\right)$ <br>  $k = 0, 1, 2$ . The three roots are:<br>  $k = 0 \rightarrow z_0 = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\$  $k=0 \rightarrow z_0 = cos\left(\frac{\pi}{2}\right) + i sin\left(\frac{\pi}{2}\right) = i;$  $\cos\left(\frac{3\pi/2 + 2k\pi}{3}\right) + i \sin\left(\frac{3\pi/2 + 2k\pi}{3}\right) = \cos\left(\frac{\pi}{2} + \frac{2}{3}k\pi\right) + i \sin\left(\frac{\pi}{2} + \frac{2}{3}k\pi\right)$ <br>  $k = 0, 1, 2$ . The three roots are:<br>  $k = 0 \rightarrow z_0 = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) = i$ ;<br>  $k = 1 \rightarrow z_1 = \cos\left(\frac{7}{6}\pi\right) + i \sin\left(\frac{$  $\frac{1}{3} \int + i \sin\left(\frac{7}{3}\right) = \cos\left(\frac{1}{2} + \frac{1}{3}k\pi\right) + i \sin\left(\frac{1}{2} + \frac{1}{3}k\pi\right)$ <br>
2. The three roots are:<br>  $z_0 = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) = i$ ;<br>  $z_1 = \cos\left(\frac{7}{6}\pi\right) + i \sin\left(\frac{7}{6}\pi\right) = -\frac{1}{2}\left(\sqrt{3} + i\right) = -\overline{z_2}$ ;<br>  $z_2 =$  $k = 0, 1, 2$ . The three roots are:<br>  $k = 0 \rightarrow z_0 = cos(\frac{\pi}{2}) + i sin(\frac{\pi}{2}) = i$ ;<br>  $k = 1 \rightarrow z_1 = cos(\frac{7}{6}\pi) + i sin(\frac{7}{6}\pi) = -\frac{1}{2}(\sqrt{3} + i) = -\overline{z_2}$ ;<br>  $k = 2 \rightarrow z_2 = cos(\frac{11}{6}\pi) + i sin(\frac{11}{6}\pi) = \frac{1}{2}(\sqrt{3} - i) = -\overline{z_1}$ .  $z_0 = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) = i;$ <br>  $z_1 = \cos\left(\frac{7}{6}\pi\right) + i \sin\left(\frac{7}{6}\pi\right) = -\frac{1}{2}\left(\sqrt{3} + i\right) = -\overline{z_2};$ <br>  $z_2 = \cos\left(\frac{11}{6}\pi\right) + i \sin\left(\frac{11}{6}\pi\right) = \frac{1}{2}\left(\sqrt{3} - i\right) = -\overline{z_1}.$ <br>
[8 k]  $k = 1 \rightarrow z_1 = \cos\left(\frac{1}{6}\pi\right) + i \sin\left(\frac{1}{6}\pi\right) = -\frac{1}{2}\left(\sqrt{3} + i\right) = -\overline{z_2};$ <br>  $k = 2 \rightarrow z_2 = \cos\left(\frac{11}{6}\pi\right) + i \sin\left(\frac{11}{6}\pi\right) = \frac{1}{2}\left(\sqrt{3} - i\right) = -\overline{z_1}.$ <br>
I M 2) The matrix  $\mathbb{A} = \begin{bmatrix} 8 & k \\ k & 8 \end{bmatrix}$  has an eigenvalue

constant. Calculate the value of k and find the matrix  $\mathbb B$  that diagonalizes matrix  $\mathbb A$ . I M 2) The matrix  $\mathbb{A} = \begin{bmatrix} 8 & k \\ k & 8 \end{bmatrix}$  has an eigenvalue  $\lambda = 4$ , where k is a positive<br>constant. Calculate the value of k and find the matrix  $\mathbb{B}$  that diagonalizes matrix  $\mathbb{A}$ .<br>If  $\lambda = 4$  is an eigenva I M 2) The matrix  $\mathbb{A} = \begin{bmatrix} 8 & k \\ k & 8 \end{bmatrix}$  has an eigenvalue  $\lambda = 4$ , where k is a positive<br>constant. Calculate the value of k and find the matrix  $\mathbb{B}$  that diagonalizes matrix  $\mathbb{A}$ .<br>If  $\lambda = 4$  is an eigenva

because k is a positive constant. To find matrix  $\mathbb B$  that diagonalizes matrix  $\mathbb A$ , we must calculate the characteristic polynomial of matrix  $\mathbb{A}$ ; = 16 - k<sup>2</sup>, put 16 - k<sup>2</sup> = 0<br>tant. To find matrix  $\mathbb{B}$  that dependence the m<br>polynomial of matrix  $\mathbb{A}$ ;<br> $\begin{vmatrix} 8 & -4 \\ 1 & \lambda - 8 \end{vmatrix} = (\lambda - 8)^2 - 16$ 

because 
$$
k
$$
 is a positive constant. To find matrix  $\mathbb{B}$  that diagonalizes matrix  $\mathbb{A}$ , we must  
start with the calculus of the second eigenvalue of the matrix  $\mathbb{A}$ , for our goal we  
calculate the characteristic polynomial of matrix  $\mathbb{A}$ ;  
 $P_{\mathbb{A}}(\lambda) = |\lambda \mathbb{I} - \mathbb{A}| = \begin{vmatrix} \lambda - 8 & -4 \\ -4 & \lambda - 8 \end{vmatrix} = (\lambda - 8)^2 - 16$ . Putting  $P_{\mathbb{A}}(\lambda) = 0$  we have  
 $(\lambda - 8)^2 = 16 \Rightarrow \lambda = 8 \pm 4$ ;  $\lambda_1 = 4$  and  $\lambda_2 = 12$ .  
*Alternative Method:* remember that the sum of all eigenvalues of a matrix is equal to the  
matrix's trace, the sum of the elements in the principal diagonal of the matrix; thus for

*Alternative Method*: remember that the sum of all eigevalues of a matrix is equal to the matrix's trace, the sum of the elements in the principal diagonal of the matrix; thus for  $P_A(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda & 1 \\ -4 & \lambda - 8 \end{vmatrix} = (\lambda - 8)^2 - 16$ . Putting  $P_A(\lambda) = 0$  we have  $(\lambda - 8)^2 = 16 \Rightarrow \lambda = 8 \pm 4$ ;  $\lambda_1 = 4$  and  $\lambda_2 = 12$ .<br>Alternative Method: remember that the sum of all eigevalues of a matrix is equal to mat ( $\lambda - \infty$ ) = 10  $\Rightarrow \lambda = \infty \pm 4$ ;  $\lambda_1 = 4$  and  $\lambda_2 = 12$ .<br>
Alternative Method: remember that the sum of all eigevalues of a matrix is equal to t<br>
matrix's trace, the sum of the elements in the principal diagonal of the ma matrix's trace, the sum of the elements in the principal diagonal of the matrix;<br>matrix A,  $\lambda_1 + \lambda_2 = 8 + 8$ , by  $\lambda_1 = 4$  trivially follows  $\lambda_2 = 12$ <br>Now we calculate the eigenvectors connected with any eigenvalue of ma ternative method. Tentender that the sum of an eigevalues of a matrix is equal to<br>atrix's trace, the sum of the elements in the principal diagonal of the matrix; thus<br>atrix  $\mathbb{A}$ ,  $\lambda_1 + \lambda_2 = 8 + 8$ , by  $\lambda_1 = 4$  trivia

Now we calculate the eigenvectors connected with any eigenvalue of matrix  $\mathbb{A}$ :  $\lambda_1 = 4$  trivially follows  $\lambda_2 = 12$ <br>ors connected with any eigenvalue of matrix A:<br>nected with  $\lambda_1$  is a vector  $(x, y)$  such that<br> $\begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$ 

$$
(4\mathbb{I} - \mathbb{A}) \cdot (x, y) = (0, 0) \Leftrightarrow \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow
$$

 $\begin{pmatrix} -4x - 4y \\ -4x - 4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x + y = 0$ ; any eigenvector connected with  $\lambda_1$  is a vector  $(x, -x)$ , for instance  $(1, -1)$ ;  $\begin{pmatrix} -4x - 4y \\ -4x - 4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x + y = 0$ ; any eigenv<br>  $(x, -x)$ , for instance  $(1, -1)$ ;<br>
2. for  $\lambda_2 = 12$  an eigenvector connected with  $\lambda_2$  is  $\begin{pmatrix} -4x - 4y \\ -4x - 4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x + y = 0$ ; any eigenvector connected with  $\lambda_1$  is a vector  $(x, -x)$ , for instance  $(1, -1)$ ;<br>2. for  $\lambda_2 = 12$  an eigenvector connected with  $\lambda_2$  is a vector  $(x, y)$  such tha  $\begin{pmatrix} -4x - 4y \\ -4x - 4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x + y = 0$ ; any eigenvector connected with  $\lambda_1$  is a vect<br>  $(x, -x)$ , for instance  $(1, -1)$ ;<br>
2. for  $\lambda_2 = 12$  an eigenvector connected with  $\lambda_2$  is a vector  $(x, y)$  such tha  $y = 0$ ; any eigenvector connected with  $\lambda_1$  is a v<br>nected with  $\lambda_2$  is a vector  $(x, y)$  such that<br> $\begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$ <br> $y = 0$ ; any eigenvector connected with  $\lambda_2$  is a v (x, -x), for instance  $(1, -1)$ ;<br>
2. for  $\lambda_2 = 12$  an eigenvector connected with  $\lambda_2$  is a vector  $(x, y)$  such that<br>  $(12\mathbb{I} - \mathbb{A}) \cdot (x, y) = (0, 0) \Leftrightarrow \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$ <br>  $(12\mathbb{I} - \mathbb{A}) \cdot (x, y) = (0, 0) \Leftrightarrow \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$ <br>  $\begin{pmatrix} 4x - 4y \\ -4x + 4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x - y = 0;$  any  $(x, x)$ , for instance  $(1, 1)$ .<br>
Matrix  $\mathbb B$  that diagonalizes matrix  $\mathbb A$  is  $\mathbb B =$  $(12\mathbb{I} - \mathbb{A}) \cdot (x, y) = (0, 0) \Leftrightarrow \begin{bmatrix} 1 & 0 \\ -4 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow$ <br>  $\begin{pmatrix} 4x - 4y \\ -4x + 4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x - y = 0$ ; any eigenvector connected with  $\lambda$ <br>  $(x, x)$ , for instance  $(1$ eigenvector connected with  $\lambda_2$ <br>  $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ .<br>  $t \mathbb{B}^{-1} \cdot \mathbb{A} \cdot \mathbb{B}$ .  $\begin{pmatrix} 1 & 1 & 0 \\ 0 & -4x + 4y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow x - y = 0$ ; any eigenvector connected with  $\lambda_2$ <br>  $(x, x)$ , for instance (1, 1).<br>
Matrix  $\mathbb{B}$  that diagonalizes matrix  $\mathbb{A}$  is  $\mathbb{B} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (*x*, *x*), for instance (1, 1).<br>
Matrix  $\mathbb{B}$  that diagonalizes matrix  $\mathbb{A}$  is  $\mathbb{B} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ .<br>
To check the result, we calculate the product  $\mathbb{B}^{-1} \cdot \mathbb{A} \cdot \mathbb{B}$ .<br>  $\mathbb{B}^{-1} \cdot \mathbb{A} \cdot \mathbb$ 

Matrix B that diagonalizes matrix A is 
$$
\mathbb{B} = \begin{bmatrix} 1 & 1 \ -1 & 1 \end{bmatrix}
$$
.  
\nTo check the result, we calculate the product  $\mathbb{B}^{-1} \cdot \mathbb{A} \cdot \mathbb{B}$ .  
\n $\mathbb{B}^{-1} \cdot \mathbb{A} \cdot \mathbb{B} = \begin{bmatrix} 1 & 1 \ -1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 & 4 \ 4 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \ -1 & 1 \end{bmatrix} =$   
\n $\frac{1}{2} \begin{bmatrix} 1 & 1 \ -1 & 1 \end{bmatrix}^T \cdot \begin{bmatrix} 4 & 12 \ -4 & 12 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 12 \ -4 & 12 \end{bmatrix} =$   
\n $\frac{1}{2} \begin{bmatrix} 8 & 0 \ 0 & 24 \end{bmatrix} = \begin{bmatrix} 4 & 0 \ 0 & 12 \end{bmatrix}$ , a diagonal matrix.  
\nIM 3) Given the linear function  $F: \mathbb{R}^3 \to \mathbb{R}^4$ , we know that for function  $F: \begin{array}{l} 1. F(1,0,0) = (0,0,0,0); \\ 2. F(1,1,0) = (1,0,0,0); \\ 3. F(1,1,1) = (1,1,0,0). \end{array}$   
\nCalculate the dimension of the kernal and the dimension of the image of  $F$  and find a

$$
1. F(1,0,0) = (0,0,0,0);
$$

1 M 3) Given the linear function  $F: \mathbb{R}^3 \to \mathbb{R}^4$ , we know that for function  $F:$ <br>
1.  $F(1,0,0) = (0,0,0,0)$ ;<br>
2.  $F(1,1,0) = (1,0,0,0)$ ;<br>
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Calculate the dimention of the kernel and the dimentio basis for both, kernel and image.

1.  $F(1, 0, 0) = (0, 0, 0, 0)$ ,<br>
2.  $F(1, 1, 0) = (1, 0, 0, 0)$ ;<br>
3.  $F(1, 1, 1) = (1, 1, 0, 0)$ .<br>
Calculate the dimention of the kernel and the dimention of the image of  $F$  and find a<br>
basis for both, kernel and image.<br>
By 2. 2.  $F(1, 1, 0) = (1, 0, 0, 0)$ ,<br>
3.  $F(1, 1, 1) = (1, 1, 0, 0)$ .<br>
Calculate the dimention of the kernel and the dimention of the image of  $F$  and find a<br>
basis for both, kernel and image.<br>
By 2. and 3. linear indipendent vect S.  $F(1, 1, 1) = (1, 1, 0, 0)$ .<br>Calculate the dimention of the kernel and the dimention of the image of  $F$  and find a<br>basis for both, kernel and image.<br>By 2. and 3. linear indipendent vectors  $(1, 0, 0, 0)$  and  $(1, 1, 0,$ greater or equal then 1, but by the dimention Theorem. By 2. and 3. linear indipendent vectors  $(1, 0, 0, 0)$  and  $(1, 1, 0, 0)$  belong to the image of function F, thus the dimention of the image is greater or equal 2; at the same time by 1. vector  $(1, 0, 0)$  belongs to the By 2. and 3. linear indipendent vectors  $(1, 0, 0, 0)$  and  $(1, 1, 0, 0)$  belong to the image of function  $F$ , thus the dimention of the image is greater or equal 2; at the same time by 1. vector  $(1, 0, 0)$  belongs to th

 $dim(Ima(F)) + dim(Ker(F)) = dim(\mathbb{R}^3) = 3$ , thus  $dim(Ima(F)) = 2$  and

yector  $(1, 0, 0)$  belongs to the kerner or function  $P$ , thus the dimention of the kerner is<br>greater or equal then 1, but by the dimention Theorem,<br> $dim(Ima(F)) + dim(Ker(F)) = dim(\mathbb{R}^3) = 3$ , thus  $dim(Ima(F)) = 2$  and<br> $dim(Ker(F)) = 1$ .<br>For the b *Alternative Solution*: by 1., 2. and 3. we can calculate the matrix  $A_F$  associated at  $(1, 0, 0, 0)$  and<br>  $xr(F) = \{(1, 0, 0)\}.$ <br>
A<sub>F</sub> associated at  $4 \times 3$ The sets, by the linear independency<br>  $\therefore B_{Ima(F)} = \{(1, 0, 0, 0), (1, 1, 0, n:$  by 1., 2. and 3. we can calculat<br>  $\begin{bmatrix} \alpha & \beta & \chi \\ \delta & F & \epsilon \\ \eta & \gamma & \iota \end{bmatrix}$ . 

For the basis of both sets, by the linear independency of vectors (1, 0, 0, 0) and  
\n(1, 1, 0, 0) we have: 
$$
B_{Ima(F)} = \{(1, 0, 0, 0), (1, 1, 0, 0)\}
$$
 and  $B_{Ker(F)} = \{(1, 0, 0)\}$ .  
\nAlternative Solution: by 1., 2. and 3. we can calculate the matrix  $\mathbb{A}_F$  associated at  
\nfunction  $F$ ,  $\mathbb{A}_F = \begin{bmatrix} \alpha & \beta & \chi \\ \delta & F & \epsilon \\ \eta & \gamma & \iota \\ \kappa & \mu & \nu \end{bmatrix}$ .  
\nFrom 1.  $F(1, 0, 0) = \begin{bmatrix} \alpha & \beta & \chi \\ \delta & F & \epsilon \\ \eta & \gamma & \iota \\ \kappa & \mu & \nu \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \delta \\ \kappa \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$   
\n $\alpha = \delta = \eta = \kappa = 0$ ;  $\mathbb{A}_F = \begin{bmatrix} 0 & \beta & \chi \\ 0 & \gamma & \iota \\ 0 & \mu & \nu \end{bmatrix}$ .

From 2. 
$$
F(1, 1, 0) = \begin{bmatrix} 0 & \beta & \chi \\ 0 & F & \epsilon \\ 0 & \gamma & \iota \\ 0 & \mu & \nu \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \beta \\ F \\ \gamma \\ \mu \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow
$$
  
\n $\beta = 1$  and  $F = \gamma = \mu = 0$ ;  $A_F = \begin{bmatrix} 0 & 1 & \chi \\ 0 & 0 & \epsilon \\ 0 & 0 & \iota \\ 0 & 0 & \iota \end{bmatrix}$ .  
\nFrom 3.  $F(1, 1, 1) = \begin{bmatrix} 0 & 1 & \chi \\ 0 & 0 & \epsilon \\ 0 & 0 & \iota \\ 0 & 0 & \nu \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + \chi \\ \epsilon \\ \iota \\ \nu \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$   
\n $\epsilon = 1$  and  $\chi = \iota = \nu = 0$ ;  $A_F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .  
\nTrivially we can notice that matrix  $A_F$  shows rank equal 2, thus the dimension of the  
\nimage of function  $F$  is 2 and the dimension of the kmell is

Trivially we can notice that matrix  $A_F$  shows rank equal 2, thus the dimention of the  $\epsilon = 1$  and  $\chi = \iota = \nu = 0$ ;  $\mathbb{A}_F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .<br>Trivially we can notice that matrix  $\mathbb{A}_F$  shows rank equal 2, thus the dimention  $F$  is 2 and the dimention of the kernel is  $dim(Ker(F$  $\epsilon = 1$  and  $\chi = \iota = \nu = 0$ ;  $\mathbb{A}_F = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .<br>Trivially we can notice that matrix  $\mathbb{A}_F$  shows rank equal 2, thus the dimention image of function F is 2 and the dimention of the kernel i . For the basis of the image, by the linear independency of the second and the third columns of matrix  $A_F$ , we can take for the basis of image the set: Filiviany we can holice that matrix  $\mathbb{A}_F$  shows rank equal 2, thus the dimention of the limage of function F is 2 and the dimention of the kernel is  $dim(Ker(F)) = dim(\mathbb{R}^3) - dim(Ima(F)) = 3 - 2 = 1$ .<br>For the basis of the image, by t belongs in the kernel of function  $F$  is 2 and the dimension of the kernel is<br>  $dim(Ker(F)) = dim(\mathbb{R}^3) - dim(Ima(F)) = 3 - 2 = 1$ .<br>
For the basis of the image, by the linear independency of the second and the third<br>
columns of matrix  $\math$ dependency of the second and<br>
dependency of the second of  $y$ <br>  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow y = z = 0$ . the image, by the linear independent<br>rix  $\mathbb{A}_F$ , we can take for the b<br>0, 0, 0, 0, (0, 1, 0, 0)}. For the<br>ternel of function F if and or<br> $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ .  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ z \\ 0 \end{pmatrix}$ 

For the basis of the image, by the linear independency of the second and the third columns of matrix 
$$
A_F
$$
, we can take for the basis of image the set:  
\n
$$
\mathcal{B}_{Ima(F)} = \{(1, 0, 0, 0), (0, 1, 0, 0)\}
$$
For the Kernel we know that a vector  $(x, y, z)$  belongs in the Kernel of function  $F$  if and only if  $F(x, y, z) = (0, 0, 0, 0)$ , but\n
$$
F(x, y, z) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y \\ z \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow y = z = 0
$$
. Thus a generic vector that belongs in the Kernel is a vector  $(x, 0, 0) = x(1, 0, 0)$ , and a basis for the Kernel is the set  $\mathcal{B}_{Ker(F)} = \{(1, 0, 0)\}$ .  
\n
$$
\begin{bmatrix} \alpha & 2 & 3 \end{bmatrix}
$$

vector that belongs in the kernel is a vector  $(x, 0, 0) = x(1, 0, 0)$ , and a basis for the kernel is the set  $\mathcal{B}_{Ker(F)} = \{(1, 0, 0)\}\.$ 

vector that belongs in the kernel is a vector  $(x, 0, 0) = x(1, 0, 0)$ , and a basis for the kernel is the set  $\mathcal{B}_{Ker(F)} = \{(1, 0, 0)\}\.$ <br>
I M 4) The matrix  $\mathbb{A} = \begin{bmatrix} \alpha & 2 & 3 \\ 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$  has determinant equal  $\begin{pmatrix} z \end{pmatrix}$   $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ <br>
el is a vector  $(x, 0, 0) =$ <br>  $\begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$  has determinant kernel is a vector  $(x, 0, 0) =$ <br>= { $(1, 0, 0)$ }.<br> $\alpha$  2 3<br>0  $\alpha$  2 has determinant Exerier is a vector  $(x, 0, 0)$ <br>
= { $(1, 0, 0)$ }.<br>  $\alpha$  2 3<br>
0  $\alpha$  2<br>
md find matrix  $\mathbb{A}^{-1}$ , the isometric following that (a)  $\begin{cases} 0 \end{cases}$ <br>
in the kernel is a vector  $(x, 0, 0) = x$ <br>  $= \{(1, 0, 0)\}.$ <br>  $\begin{bmatrix} \alpha & 2 & 3 \\ 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$  has determinant equals to  $\alpha$  $=\{(1,0,0)\}\.$ <br>  $=\{(1,0,0)\}\.$ <br>  $\begin{bmatrix} \alpha & 2 & 3 \\ 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$  has determinant equals and find matrix  $\mathbb{A}^{-1}$ , the inverse  $\alpha$  2 3]  $\alpha$  2 | has d  $\alpha$ has determinant equal 8, where  $\alpha$  is a positive constant. IM 4) The matrix  $A = \begin{bmatrix} \alpha & 2 & 3 \\ 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$  has determinant equal 8, where  $\alpha$  is a positive constant.<br>Calculate the value of  $\alpha$  and find matrix  $A^{-1}$ , the inverse matrix of matrix A.<br>Matrix A is an up I M 4) The matrix  $A = \begin{bmatrix} \alpha & 2 & 3 \\ 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$  has determinant equal 8, where  $\alpha$  is a positive constant.<br>Calculate the value of  $\alpha$  and find matrix  $A^{-1}$ , the inverse matrix of matrix  $A$ .<br>Matrix  $A$  is

Calculate the value of  $\alpha$  and find matrix  $\mathbb{A}^{-1}$ , the inverse matrix of matrix  $\mathbb{A}$ . I M 4) The matrix  $\mathbb{A} = \begin{bmatrix} \alpha & 2 & 3 \\ 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$  has determinant equal 8, where  $\alpha$  is a positive constant<br>Calculate the value of  $\alpha$  and find matrix  $\mathbb{A}^{-1}$ , the inverse matrix of matrix  $\mathbb{A}$ .<br> Calculate the value of  $\alpha$  and find matrix  $\mathbb{A}^{-1}$ , the inverse matrix of matrix  $\mathbb{A}$ .<br>Matrix  $\mathbb{A}$  is an up-triangular matrix, following that the determinat of  $\mathbb{A}$  is the its elements in the principal di The matrix  $A = \begin{bmatrix} 0 & \alpha & 2 \\ 0 & 0 & \alpha \end{bmatrix}$ <br>te the value of  $\alpha$  and find m<br>A is an up-triangular matrix<br>nents in the principal diagonal<br> $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ . Remember that  $\begin{bmatrix} 0 & 0 & \alpha \end{bmatrix}$ <br>alate the value of  $\alpha$  and find mat<br>ix A is an up-triangular matrix, f<br>ements in the principal diagonal:<br> $\begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ . Remember that th

Calculate the value of 
$$
\alpha
$$
 and find matrix A<sup>\*</sup>, the inverse matrix of matrix A.  
\nMatrix A is an up-triangular matrix, following that the determinant of A is the product of  
\nits elements in the principal diagonal:  $|A| = \alpha^3$ , put  $\alpha^3 = 8$  we get  $\alpha = 2$ ;  
\n
$$
A = \begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}
$$
. Remember that the inverse matrix of A is the matrix  
\n
$$
A^{-1} = \frac{1}{|A|} (Adj(A))^T
$$
, where  $Adj(A)$  is the adjoin matrix of A.  
\n
$$
A^{-1} = \frac{1}{8} \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}^T = \frac{1}{8} \begin{bmatrix} 4 & 0 & 0 \\ -4 & 4 & 0 \\ -2 & -4 & 4 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 \\ 2 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}
$$

$$
\frac{1}{8} \begin{bmatrix} 4 & -4 & -2 \\ 0 & 4 & -4 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & -1/4 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix}.
$$
  
To check the result, we calculate the product  $\mathbb{A}^{-1} \cdot \mathbb{A}$ .  
 $\mathbb{A}^{-1} \cdot \mathbb{A} = \begin{bmatrix} 1/2 & -1/2 & -1/4 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , an identity matrix.  
  
\nII M 1) Solve the problem  $\begin{cases} \text{Max/min } f(x, y) = 4x + 4y \\ \text{u.c.: } x^2 + y^2 \le 4 \end{cases}$ .  
  
\nThe function f is an affine continuous function, the admissible region is a circle with  
\ncenter (0, 0) and radius equal 9, is undefined by the  
\nshorter (0, 0) and radius equal 9, and also get the before f represents the  
\n*h*

matrix.

II M 1) Solve the problem 
$$
\begin{cases} \text{Max/min } f(x, y) = 4x + 4y \\ u.c. : x^2 + y^2 \le 4 \end{cases}.
$$

II M 1) Solve the problem  $\begin{cases} \text{Max/min } f(x, y) = 4x + 4y \\ \text{u.c.: } x^2 + y^2 \le 4 \end{cases}$ .<br>The function f is an affine continuos function, the admissible region is a circle with center (0, 0) and radius equal 2, a bounded and closed s maximum and minimum in the admissible region. The Lagrangian function is  $\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4)$  with II M 1) Solve the problem  $\begin{cases} \text{max/min } f(x, y) = 4x + 4y \\ u.c.: x^2 + y^2 \le 4 \end{cases}$ .<br>The function f is an affine continuos function, the admissible center (0, 0) and radius equal 2, a bounded and closed set, the maximum and minimum The function f is an affine continuos function, the admissible reg<br>center (0, 0) and radius equal 2, a bounded and closed set, therefor<br>maximum and minimum in the admissible region. The Lagrangia<br> $\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda$ The function f is an affine continuos function, the<br>center (0, 0) and radius equal 2, a bounded and clo<br>maximum and minimum in the admissible region.<br> $\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4)$  with<br> $\nabla \mathcal{L} = (4 - 2\lambda x, 4 - 2\lambda y$  $\bigwedge \lambda = 0$  $\left\{\begin{array}{cc} 1 & 0 \\ 4 & -1 \end{array}\right.$   $S$  $4 = 0$  $x^2+y^2\leq 4$ where  $(0, 0)$  and radius equality<br>  $x, y, \lambda) = 4x + 4y - \lambda(x)$ <br>  $\mathcal{L} = (4 - 2\lambda x, 4 - 2\lambda y,$ <br>  $CASE$  (free optimizat<br>  $\lambda = 0$ <br>  $4 = 0$  System imp  $(x)$ <br>  $(x, y, \lambda) = 4x + 4y - \lambda(x)$ <br>  $\mathcal{L} = (4 - 2\lambda x, 4 - 2\lambda y,$ <br>  $CASE$  (free optimizar<br>  $\lambda = 0$ <br>  $4 = 0$  System imp<br>  $4 = 0$  $x, y, \lambda$ ) =  $4x + 4y - \lambda(3)$ <br>  $\mathcal{L} = (4 - 2\lambda x, 4 - 2\lambda y,$ <br>  $CASE$  (free optimizar<br>  $\lambda = 0$ <br>  $4 = 0$  System imp<br>  $x^2 + y^2 \le 4$  $CASE$  (free optimization):<br>  $\lambda = 0$ <br>  $4 = 0$  System impossible<br>  $x^2 + y^2 \le 4$ <br>  $CASE$  (constrained optimical) . System impossible.  $\begin{cases}\n\lambda = 0 \\
4 = 0 \\
4 = 0\n\end{cases}$ . System impossible.<br>  $\begin{cases}\n\lambda = 0 \\
4 = 0 \\
x^2 + y^2 \le 4 \\
II^{\circ} \ \ \text{CASE} \ \text{(constrained optimization)}:\n\begin{cases}\n\lambda \neq 0 \\
0\n\end{cases}$  $\begin{cases} 4 = 0 & . \text{ System impossible.} \\ 4 = 0 & . \end{cases}$ <br>  $\begin{cases} \lambda \neq 0 \\ 4 - 2\lambda x = 0 \\ 4 - 2\lambda y = 0 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{$  $\begin{cases} x^2 + y^2 \leq 4 \\ I^1 \circ \text{CASE} \text{ (constrained optimization)}: \\ 4 - 2\lambda x = 0 \\ 4 - 2\lambda y = 0 \end{cases}$   $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ ( \frac{2}{\lambda})^2 + (\frac{2}{\lambda})^2 = 4 \end{cases}$   $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ \frac{8}{\lambda^2} = 4 \end{cases}$   $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\$  $I^{\circ}$  CASE (constrained optimization):<br>  $\begin{cases} \lambda \neq 0 \\ 4 - 2\lambda x = 0 \\ 4 - 2\lambda y = 0 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ (\frac{2}{\lambda})^2 + (\frac{2}{\lambda})^2 = 4 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac$ If CASE (constrained optimization):<br>  $\begin{cases} \lambda \neq 0 \\ 4 - 2\lambda x = 0 \\ x^2 + y^2 = 4 \end{cases}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$  tem impossible.<br>
cained optimization):<br>  $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2} \\ y = \pm \sqrt{2} \end{cases}$ rained optimization):<br>  $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ (\frac{2}{\lambda})^2 + (\frac{2}{\lambda})^2 = 4 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2} \\ y = \pm \sqrt{2} \\ \lambda^2 = 2 \end{cases}$ , dined optimization).<br>  $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ \left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 = 4 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ \frac{8}{\lambda^2} = 4 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ \lambda^2 = 2 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2}$  $\lambda \neq 0$  $\lambda x = 0$  $\lambda y = 0$  $\lambda \neq 0$   $\lambda \neq 0$   $\lambda \neq 0$  $\lambda^2=2$ = 0<br>
= 0<br>  $(ASE (constrained optimization))$ <br>  $\neq 0$ <br>  $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ 0 \end{cases}$ <br>  $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ x = \frac{2}{\lambda} \end{cases}$ <br>  $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ x = \frac{2}{\lambda} \end{cases}$ <br>  $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ x = \frac{2}{\lambda} \end{cases}$ <br>  $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \$  $4 = 0$  System impossible<br>  $x^2 + y^2 \le 4$ <br>  $\circ$  CASE (constrained optimix<br>  $\lambda \ne 0$ <br>  $4 - 2\lambda x = 0$ <br>  $4 - 2\lambda y = 0$ <br>  $\Rightarrow$   $\begin{cases} \lambda \ne 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$  $x^2 + y^2 \le 4$ <br>
° CASE (constrained optimi.<br>  $\lambda \ne 0$ <br>  $4 - 2\lambda x = 0$ <br>  $4 - 2\lambda y = 0$   $\Rightarrow$   $\begin{cases} \lambda \ne 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ \left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 \end{cases}$ ° CASE (constrained optimi<br>  $\lambda \neq 0$ <br>  $4 - 2\lambda x = 0$ <br>  $x = \frac{2}{\lambda}$ <br>  $x^2 + y^2 = 4$ <br>  $x = \frac{2}{\lambda}$ <br>  $\left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)$ <br>  $\left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)$ <br>  $\left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)$ <br>  $\left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)$ <br>  $\left(\frac{$ impossible.<br>
ed optimization):<br>  $\neq 0$ <br>  $=\frac{2}{\lambda}$ <br>  $\Rightarrow$   $\begin{cases} \lambda \\ x \end{cases}$ m impossible.<br>
ned optimization):<br>  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$ <br>  $y = \frac{2}{\lambda}$   $\Rightarrow$ ned optimization):<br>  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$ <br>  $y = \frac{2}{\lambda}$   $\Rightarrow$ <br>  $(\frac{2}{\lambda})^2 + (\frac{2}{\lambda})^2 = 4$ pptimization):<br>  $\frac{2}{\lambda}$ <br>  $\frac{2}{\lambda}$ <br>  $\Rightarrow$ <br>  $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$ <br>  $\Rightarrow$ <br>  $\frac{8}{\lambda^2} = 4$ <br>
<br>
ts  $P_1(\sqrt{2}, \sqrt{2})$ ,  $P_2(-\sqrt{2}, \sqrt{2})$  $\neq 0$ <br>=  $\frac{2}{\lambda}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \end{cases}$  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$ <br>  $y = \frac{2}{\lambda}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$ <br>  $\frac{8}{\lambda^2} = 4$   $\lambda^2 = 2$  $\begin{array}{l} \neq 0 \\ = \frac{2}{\lambda} \\ = \frac{2}{\lambda} \end{array}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$   $\Rightarrow$   $\begin{cases} x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases}$   $\Rightarrow$   $\begin{cases} (-\sqrt{2}, -\sqrt{2}) \end{cases}$ . The  $\neq 0$ <br>=  $\frac{2}{\lambda}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2} \end{cases}$  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2} \\ y = \frac{2}{\lambda} \end{cases}$  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$ <br>  $y = \frac{2}{\lambda}$   $\Rightarrow$   $\begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2} \\ y = \pm \sqrt{2} \\ \lambda = \pm \sqrt{2} \end{cases}$  $\begin{cases}\n\frac{2}{\lambda} = \frac{2}{\lambda} \\
\frac{2}{\lambda} = \frac{2}{\lambda} \\
\end{cases}$   $\Rightarrow$   $\begin{cases}\n\lambda \neq 0 \\
x = \pm \sqrt{2} \\
y = \pm \sqrt{2} \\
\lambda = \pm \sqrt{2}\n\end{cases}$ <br>  $\Rightarrow$   $\sqrt{2}$ ). The first point  $\lambda \neq 0$ <br>  $x = \pm \sqrt{2}$ .  $\neq 0$ <br>  $-2\lambda x = 0$ <br>  $-2\lambda y = 0$ <br>  $y^2 + y^2 = 4$ <br>  $\left(\begin{array}{c}\lambda \\ x \\ y \\ z\end{array}\right)$  $\frac{2}{ }$  $\frac{2}{2}$  $\neq 0$ <br>  $= \frac{2}{\lambda}$ <br>  $= \frac{2}{\lambda}$ <br>  $\leftarrow (\frac{2}{\lambda})^2 = 4$ <br>  $\leftarrow (\frac{2}{\lambda})^2$ <br>  $\leftarrow (\frac{2}{\lambda})^2$  $\frac{2}{ }$  |  $\frac{2}{ }$   $\rightarrow$   $\circ$  $\frac{8}{ } - 4$  $\frac{2}{3}$  $\frac{2}{ }$   $\rightarrow$   $\rightarrow$ sstrained optimization):<br>  $\Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \pm \sqrt{2} \\ y = \pm \sqrt{2} \end{cases};$  $\lambda$  $\lambda$ =  $\frac{2}{\lambda}$ <br>
=  $\frac{2}{\lambda}$ <br>  $\frac{2}{\lambda}$ )<sup>2</sup> + ( $\frac{2}{\lambda}$ )<sup>2</sup> = 4<br>
points  $P_1(\sqrt{2}, \sqrt{2})$ ,  $\lambda \rightarrow$  )  $\lambda$  and  $\lambda$  $\lambda^2$  –  $\tau$  $\lambda \rightarrow$  )  $\lambda$  | m impossible.<br>  $\lambda \neq 0$ <br>  $x = \frac{2}{\lambda}$ <br>  $y = \frac{2}{\lambda}$ <br>  $(\frac{2}{\lambda})^2 + (\frac{2}{\lambda})^2 = 4$ <br>  $\lambda = \frac{2}{\lambda}$ <br>  $\lambda \neq 0$ <br>  $y = \frac{2}{\lambda}$ <br>  $\lambda^2 = 2$ <br>  $\lambda = \pm \sqrt{2}$ <br>  $\lambda = \pm \sqrt{2}$ <br>  $\lambda =$  $\lambda \neq 0$ <br>  $x = \pm \sqrt{2}$ <br>  $y = \pm \sqrt{2}$ ;<br>  $\lambda = \pm \sqrt{2}$  $\neq 0$ <br>  $= \pm \sqrt{2}$ <br>  $= \pm \sqrt{2}$ <br>  $= \pm \sqrt{2}$ <br>
st point  $\sqrt{2}$  $\sqrt{2}$ ,  $\lambda = \pm \sqrt{2}$ ;  $\begin{cases} \lambda \neq 0 \\ 4 - 2\lambda x = 0 \\ 4 - 2\lambda y = 0 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \\ (\frac{2}{\lambda})^2 + (\frac{2}{\lambda})^2 = 4 \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \lambda \neq 0 \\ x = \frac{2}{\lambda} \\ y = \frac{$  $\begin{cases} \frac{4-2\lambda x-6}{4-2\lambda y=0} \Rightarrow \begin{cases} y=\frac{2}{\lambda} \\ y=\frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \frac{x}{y}=\frac{\lambda}{\lambda} \Rightarrow \begin{cases} \frac{x}{y}=\frac{\lambda}{\lambda} \\ y=\frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \frac{x}{y}=\frac{\lambda}{\lambda} \\ \frac{8}{\lambda^2}=4 \end{cases} \Rightarrow \begin{cases} \frac{x}{y}=\frac{\lambda}{\lambda} \\ \frac{x}{y}=\frac{2}{\lambda} \end{cases} \Rightarrow \begin{cases} \frac{x}{y}=\frac{\lambda}{\lambda} \\ y=\pm\sqrt$  $\left(x^2 + y^2 = 4\right) \left(\frac{2}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 = 4 \left(\frac{8}{\lambda^2} - 4\right) \left(\frac{8}{\lambda^2} - 2\right)$ . The first point<br>two constrained critical points  $P_1(\sqrt{2}, \sqrt{2})$ ,  $P_2(-\sqrt{2}, -\sqrt{2})$ . The first point<br>presents  $\lambda > 0$ , point of maximum, presents  $\lambda > 0$ , point of maximum, the second presents  $\lambda < 0$ , point of mimimum. We<br>get the maximum  $f(\sqrt{2}, \sqrt{2}) = 8\sqrt{2}$  and the minimum<br> $f(-\sqrt{2}, -\sqrt{2}) = -8\sqrt{2}$ .<br>Alternative Solution: The function f is an affine contin

implies that the point of maximum and the point of minimum must be found on the border of the admissible region. We write the Lagrangian function of the problem as<br>  $\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4)$  with<br>  $\nabla \mathcal{L} = (4 - 2\lambda x, 4 - 2\lambda y, -(x^2 + y^2 - 4)).$ <br>  $FOC:$ <br>  $\begin{cases} 4 - 2\lambda x = 0 \\ 4 - 2\lambda y = 0 \end{cases}$   $\Rightarrow \begin$ Alternative Solution: The function f is an affine continuos fively and the point of maximum and the point of minimum border of the admissible region. We write the Lagrangian function  $\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4)$  *Alternative Solution:* The function *J* is an armie continuos function<br>region is a circle with center (0, 0) and radius equal 2, a bounded<br>implies that the point of maximum and the point of minimum mu<br>border of the admis region is a circle with center (0, 0) and radius equal<br>implies that the point of maximum and the point of<br>border of the admissible region. We write the Lagr<br> $\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4)$  with<br> $\nabla \mathcal{L} = (4 - 2\lambda x,$ Since that the point of maximum and the point of minimum must be found on the<br>
der of the admissible region. We write the Lagrangian function of the problem<br>  $(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4)$  with<br>  $\lambda^2 = (4 - 2\lambda x, 4 - 2\lambda y, -($ region. We write the I<br>  $\lambda(x^2 + y^2 - 4)$  with<br>  $\lambda y$ ,  $-(x^2 + y^2 - 4)$ ).<br>  $x = \frac{2}{\lambda}$ <br>  $y = \frac{2}{\lambda}$   $\Rightarrow$ on of the problem as<br>  $x = \pm \sqrt{2}$ <br>  $y = \pm \sqrt{2}$ ; two

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\mathcal{L}(x, y, \lambda) = 4x + 4y - \lambda(x^2 + y^2 - 4) \text{ with}
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\n
$$
\nabla \mathcal{L} = (4 - 2\lambda x, 4 - 2\lambda y, -(x^2 + y^2 - 4)).
$$
\n
$$
\text{FOC:}
$$
\n
$$
\begin{cases}\n4 - 2\lambda x = 0 \\
4 - 2\lambda y = 0 \Rightarrow \begin{cases}\nx = \frac{2}{\lambda} \\
y = \frac{2}{\lambda} \\
x^2 + y^2 = 4\n\end{cases} \Rightarrow \begin{cases}\nx = \frac{2}{\lambda} \\
y = \frac{2}{\lambda} \\
\frac{8}{\lambda^2} = 4\n\end{cases} \Rightarrow \begin{cases}\nx = \pm\sqrt{2} \\
y = \pm\sqrt{2} \\
y = \pm\sqrt{2}\n\end{cases}; \text{ two}
$$
\n
$$
\text{constraint critical points } P_{1,2} = \left(\pm\sqrt{2}, \pm\sqrt{2}\right).
$$
\n
$$
\text{SOC:}
$$
\n
$$
\overline{\mathcal{H}} = \begin{bmatrix}\n0 & -2x & -2y \\
-2x & -2\lambda & 0 \\
0 & 0 & 2\n\end{bmatrix}, \text{ with } |\overline{\mathcal{H}}| = \begin{bmatrix}\n0 & -2x & -2y \\
-2x & -2\lambda & 0 \\
0 & 0 & 2\n\end{bmatrix} = \begin{bmatrix}\n0 & -2x & -2y \\
-2x & -2\lambda & 0 \\
0 & 0 & 2\n\end{bmatrix} = \begin{bmatrix}\n0 & -2x & -2y \\
-2x & -2\lambda & 0 \\
0 & 0 & 2\n\end{bmatrix}
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SOC:
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\begin{aligned}\n\left( x^2 + y^2 = 4 \right) \quad & \left( \left( \frac{2}{\lambda} \right)^2 + \left( \frac{2}{\lambda} \right)^2 = 4 \right) \quad \text{Consider } \mathbb{R} = 4 \quad \text{where } \mathbb{R} = 4 \quad \text{
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$$
-2\lambda \cdot \begin{vmatrix} 0 & -2x \\ -2x & -2\lambda \end{vmatrix} - 2y \cdot \begin{vmatrix} -2x & -2y \\ -2\lambda & 0 \end{vmatrix} = 8\lambda x^2 + 8\lambda y^2 = 8\lambda (x^2 + y^2).
$$
  
\n
$$
|\overline{\mathcal{H}}(P_1)| = 32\sqrt{2} > 0, P_1 \text{ point of maximum with } f(P_1) = 8\sqrt{2},
$$
  
\n
$$
|\overline{\mathcal{H}}(P_2)| = -32\sqrt{2} < 0, P_2 \text{ point of minimum with } f(P_2) = -8\sqrt{2}.
$$
  
\nII M 2) Given the equation  $(x^8 + y^8 + z^8) - (x^6 + y^6 + z^6) = 0$  satisfied at the point  $P(1, 1, -1)$ ; verify that with it an implicit function  $(x, y) \mapsto z(x, y)$  can be defined and then calculate. for this implicit function, the partial derivatives  $z'$  and  $z'$ .

 $|\overline{\mathcal{H}}(P_1)| = 32\sqrt{2} > 0$ ,  $P_1$  point of maximum with  $f(P_1) = 8\sqrt{2}$ ,<br>  $|\overline{\mathcal{H}}(P_2)| = -32\sqrt{2} < 0$ ,  $P_2$  point of minimum with  $f(P_2) = -8\sqrt{2}$ .<br>
II M 2) Given the equation  $(x^8 + y^8 + z^8) - (x^6 + y^6 + z^6) = 0$  satisfied  $|\overline{\mathcal{H}}(P_2)| = -32\sqrt{2} < 0$ ,  $P_2$  point of minimum with  $f(P_2) = -8\sqrt{2}$ .<br>
II M 2) Given the equation  $(x^8 + y^8 + z^8) - (x^6 + y^6 + z^6) = 0$  satisfied at the point  $P(1, 1, -1)$ ; verify that with it an implicit function  $(x, y) \mapsto$ If M 2) Given the equation  $(x^8 + y^8 + z^8) - (x^6 + y^6 + z^6) = 0$  satisfied at the point<br>  $P(1, 1, -1)$ ; verify that with it an implicit function  $(x, y) \mapsto z(x, y)$  can be defined<br>
and then calculate, for this implicit function, the II M 2) Given the equation  $(x^8 + y^8 + z^8) - (x^6 + y^6 + z^6) = 0$  satisfied at the point  $P(1, 1, -1)$ ; verify that with it an implicit function  $(x, y) \mapsto z(x, y)$  can be defined and then calculate, for this implicit function, the pa  $P(1, 1, -1)$ ; verify that with it an implicit function  $(x, y) \mapsto z(x, y)$  can be defined<br>and then calculate, for this implicit function, the partial derivatives  $z'_x$  and  $z'_y$ .<br>Consider the function  $f: \mathbb{R}^3 \to \mathbb{R}$  with and then calculate, for this implicit function, the partial derivatives  $z'_x$  and  $z'_y$ .<br>Consider the function  $f: \mathbb{R}^3 \to \mathbb{R}$  with  $f(x, y, z) = (x^8 + y^8 + z^8) - (x^6 + y^6 + z^6)$ <br> $f(P) = 3 - 3 = 0$  and  $f$  is differentiable at an Consider the function  $f: \mathbb{R}^3 \to \mathbb{R}$  with  $f(x, y, z) = (x^8 + y^8 + z^8) - (x^6 + y^6 + z^6)$ .<br>  $f(P) = 3 - 3 = 0$  and f is differentiable at any point  $(x, y, z)$ , the partial derivative of<br>
f respect the variable z is  $f'_z = 8z^7 - 6z^5$ partial derivatives  $z'_x$  and  $z'_y = 8z^7 - 6z^5$ ; the proposed equation defines in a<br>partial derivative of  $f$  respect the variable  $z$  is  $f'_z = 8z^7 - 6z^5$ ; at point  $P(1, 1, -1)$  the partial derivative has value  $f'_z(P) = -8$ f respect the variable z is  $f'_z = 8z^7 - 6z^5$ ; at point  $P(1, 1, -1)$  the partial derivative<br>has value  $f'_z(P) = -8 + 6 = -2 \neq 0$ ; the proposed equation defines in a<br>neighbourhood of point P an implicit function  $(x, y) \mapsto z(x, y)$ . has value  $f_z(P) = -8 + 6 = -2 \neq 0$ ; the proposed equation defines in a<br>neighbourhood of point P an implicit function  $(x, y) \mapsto z(x, y)$ . To calculate the<br>partial derivatives  $z'_x$  and  $z'_y$  we must firstly calculate the two parti proposed equation definition (x, y)  $\mapsto z(x, y)$ . To<br>calculate the two partial<br> $f'_x(P) = f'_y(P) = 2$ . Therefore  $f'_x(P) = 1$  and  $z'_y(1, 1)$ on  $(x, y) \mapsto z(x, y)$ . To calculate the<br>calculate the two partial derivatives  $f'_x$  and<br> $f'_x(P) = f'_y(P) = 2$ . The two partial<br> $f'_x(P) = 1$  and  $z'_y(1, 1) = -\frac{f'_y(P)}{f'_z(P)} = 1$ .<br> $\pm y$  calculate the second order directional  $-2 \neq 0$ ; the proposed equation defines in a<br>
mplicit function  $(x, y) \mapsto z(x, y)$ . To calculate the<br>
e must firstly calculate the two partial derivatives  $f'_x$  and<br>  $f' - 6y^5$ ; with  $f'_x(P) = f'_y(P) = 2$ . The two partial<br>  $f'_x(1,$ partial derivatives  $z'_x$  and  $z'_y$  we must firstly calculate the two partial derivatives  $f'_x$  and<br>  $f'_y$ :  $f'_x = 8x^7 - 6x^5$ ;  $f'_y = 8y^7 - 6y^5$ ; with  $f'_x(P) = f'_y(P) = 2$ . The two partial<br>
derivatives of function  $z$  are  $z'_$  $-2 \neq 0$ ; the proposed equation defines in a<br>
uplicit function  $(x, y) \mapsto z(x, y)$ . To calculate the<br>
must firstly calculate the two partial derivatives  $f'_x$  and<br>  $-6y^5$ ; with  $f'_x(P) = f'_y(P) = 2$ . The two partial<br>  $(1, 1) = -\frac{$  $(x, y) \mapsto z(x, y)$ . To calculate the<br>
lculate the two partial derivatives  $f'_x$  and<br>  $f'_x(P) = f'_y(P) = 2$ . The two partial<br>  $\frac{(P)}{(P)} = 1$  and  $z'_y(1, 1) = -\frac{f'_y(P)}{f'_z(P)} = 1$ .<br>  $y)$ , calculate the second order directional the<br>s  $f'_x$  and<br>tial<br> $\frac{(P)}{(P)} = 1$ .

derivatives of function z are  $z'_x(1,1) = -\frac{f'_x(P)}{f'_z(P)} = 1$  and  $z'_y(1,1) = -\frac{f'_y(P)}{f'_z(P)} = 1$ .<br>
II M 3) Given the function  $f(x, y) = cos(x + y)$ , calculate the second order directional derivative  $\mathcal{D}_{v,w}^{(2)} f(0,0)$ ; where v is in the function  $f$ <br> $_{v,w}^{(2)} f(0,0)$ ; where II M 3) Given the function  $f(x, y) = c$ <br>derivative  $\mathcal{D}_{v,w}^{(2)} f(0,0)$ ; where v is the u<br> $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ .<br>Function f is differentiable at any poin Given the function  $f(x, y) =$ <br>  $ve \mathcal{D}_{v,w}^{(2)} f(0,0)$ ; where v is the<br>  $\frac{2}{2}, -\frac{\sqrt{2}}{2}$ . Given the function  $f(x, y) =$ <br>ive  $\mathcal{D}_{v,w}^{(2)} f(0, 0)$ ; where v is the<br> $\left(\frac{2}{2}, -\frac{\sqrt{2}}{2}\right)$ .<br>on f is differentiable at any po derivative  $\mathcal{D}_{v,w}^{(2)} f(0,0)$ ; where v is the unit vector  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$  and w is<br>  $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ .<br>
Function f is differentiable at any point  $(x, y)$  with gradient vector<br>  $\nabla f(x, y) = (-\operatorname{sen}(x + y),$ 

$$
\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right).
$$

 $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ .<br>
Function f is differentiable at any point  $(x, y)$  with gradient vector<br>  $\nabla f(x, y) = (-\operatorname{sen}(x + y), -\operatorname{sen}(x + y))$  and hessian matrix<br>  $\mathcal{H}f(x, y) = \begin{bmatrix} -\cos(x + y) & -\cos(x + y) \\ -\cos(x + y) & -\cos(x + y) \end{bmatrix}$  with  $\$  $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ .<br>
Function f is differentiable at any point  $(x, y)$  with gradient vector<br>  $\nabla f(x, y) = (-\operatorname{sen}(x + y), -\operatorname{sen}(x + y))$  and hessian matrix<br>  $\mathcal{H}f(x, y) = \begin{bmatrix} -\cos(x + y) & -\cos(x + y) \\ -\cos(x + y) & -\cos(x + y) \end{bmatrix}$  with  $\$ with  $\mathcal{H}f(0,0) = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ .  $\mathcal{D}^{(2)}_{v,w}f(0,0)=$  |  $f(x,y) = \begin{bmatrix} -c \ -c \ \frac{c^2}{v,w} f(0,0) = \begin{pmatrix} -c \ \frac{c^2}{w} \end{pmatrix}$  $\sqrt{2}$  $\sqrt{2}$  |  $\sqrt{2}$  $\chi_{v,w}^{(2)}f(0,0)=$ 2  $\big\}$ 2 |  $-$ 2  $\vert$  $-\frac{\sqrt{2}}{2}$ tion *f* is differentiable at any point  $(x, y)$  with gradient vector<br>  $x, y$  =  $(-\operatorname{sen}(x + y), -\operatorname{sen}(x + y))$  and hessian matrix<br>  $x, y$  =  $\begin{bmatrix} -\cos(x + y) & -\cos(x + y) \\ -\cos(x + y) & -\cos(x + y) \end{bmatrix}$  with  $\mathcal{H}f(0, 0) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ <br>  $f(0,$ on f is differentiable at any point  $(x, y)$  with gradient vector<br>  $y$ ) =  $(-\operatorname{sen}(x + y), -\operatorname{sen}(x + y))$  and hessian matrix<br>  $y$ ) =  $\begin{bmatrix} -\cos(x + y) & -\cos(x + y) \\ -\cos(x + y) & -\cos(x + y) \end{bmatrix}$  with  $\mathcal{H}f(0, 0) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ <br>  $(0, 0) = \left(\$  $\mathcal{D}_{v,w}^{(2)} f(0,0) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \mathcal{H} f(0,0) \cdot \left(-\frac{\sqrt{2}}{2}\right) =$ <br>  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left[-\frac{1}{2} - \frac{1}{2}\right] \cdot \left(-\frac{\sqrt{2}}{2}\right) =$ <br>  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left[-\frac{1}{2} - \frac{1}{2}\right] \cdot \left(-\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2$ (c, g)  $[-\cos(x+y) - \cos(x+y)]$  and  $\lim_{x \to 0} f(x, y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \mathcal{H}f(0, 0) \cdot \left(-\frac{\sqrt{2}}{2}\right) =$ <br>  $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$   $\left(-\frac{1}{2} - \frac{1}{2}\right) \cdot \left(-\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left(\sqrt{2}, \sqrt{2}\right) = 2.$ 3)  $[-\cos(x+y) - \cos(x+y)]$  and  $\cos(x, y)$   $[-1 -1]$ <br>
(0, 0)  $= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \mathcal{H}f(0,0) \cdot \left(-\frac{\sqrt{2}}{2}\right) =$ <br>  $\left[\frac{\sqrt{2}}{-1}, \frac{-1}{-1}\right] \cdot \left(-\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left(\sqrt{2}, \sqrt{2}\right) = 2.$  $\begin{split} &-\cos(x+y) \quad -\cos(x+y) \, \Bigg[ \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \Bigg) \cdot \mathcal{H} f(0,0) \cdot \Bigg( \, -\frac{\sqrt{2}}{2} \, \Bigg) \ &\quad \Bigg[ \, -1 \quad -1 \, \Bigg] \cdot \Bigg( \, -\frac{\sqrt{2}}{2} \, \Bigg) \, = \, \Bigg( \, \frac{\sqrt{2}}{2} \, \Bigg) \end{split}$   $\sqrt{2}$  / /  $\sqrt{2}$  |  $\sqrt{2}$  | 2 2  $\big\}$ 2 |  $-$ 2  $\vert$ 2  $/$  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left[-\frac{1}{2}, -\frac{1}{2}\right] \cdot \left(-\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left(\sqrt{2}, \sqrt{2}\right) = 2.$ <br>II M 4) Calculate the partial derivatives of function  $f(x, y, z, w) = e^{x+2y} - 3zw^3$ .<br> $f'_x = e^{x+2y}$   $f'_y = 2e^{x+2y}$  $\begin{aligned} \frac{\sqrt{2}}{2} \Bigg) \cdot \left[ \begin{array}{cc} -1 & -1 \\ -1 & -1 \end{array} \right] \cdot \left( \begin{array}{c} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{array} \right) &= \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \cdot \left( \sqrt{2}, \sqrt{2} \right) = 2. \end{aligned}$ <br>
alculate the partial derivatives of function  $f(x, y, z, w) = e^{x+2y} - 3zw^3$ 

 $\int |x - 1| = \int |x - \frac{\sqrt{2}}{2}|$   $\int |x - \sqrt{2}| = C$ <br>
culate the partial derivatives of function  $f(x, y, z, w) = e^{x + 2y} - 3zw^3$ .<br>  $f'_x = e^{x + 2y}$   $f'_y = 2e^{x + 2y}$   $f'_z = -3w^3$   $f'_w = -9zw^2$ . .

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