UNIVERSITÀ DEGLI STUDI DI SIENA
Scuola di Economia e Management
A.A. 2025/26
Quantitative Methods for Economic Applications - Mathematics for Economic Applications
Task 11/6/2026

I M 1) Find the roots of the equation $x^4 + 1 = 0$ and then calculate the sum and the product of the two roots with positive real part.

I M 2) Consider the matrix $\mathbb{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. Calculate its eigenvalues and study if the matrix $\mathbb{A}$ is a diagonalizable one.

I M 3) Given a linear map $F: \mathbb{R}^3 \to \mathbb{R}^4$, with $F(x_1, x_2, x_3) = (0, ax_1, bx_1 + x_2, cx_1 + x_2 + x_3)$; we know that $F(1, 1, 1) = (0, 2, 4, 8)$. Calculate the values of the parameters a, b and c and then find a basis for the image of linear map F.

I M 4) Given the matrix $\mathbb{X} = \begin{bmatrix} \frac{1}{2} & \alpha \\ \alpha & -\frac{1}{2} \end{bmatrix}$, find the values of parameter α knowing that matrix $\mathbb{X}$ is an orthogonal matrix, and with the values of α found calculate the determinant of matrix $\mathbb{X}$.

II M 1) Given the equation $f(x, y, z) = e^{x+y-z} - 2xyz - z = 0$ satisfied at the point $(1, 0, 1)$, verify that with it an implicit function $y = y(x, z)$ can be defined and then calculate, for this implicit function its gradient vector $\nabla y(1, 1)$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x + xy \\ \text{u.c.: } x^2 + 4y^2 \leq 4 \end{cases}$.

II M 3) Given the vector value function $f: \mathbb{R} \to \mathbb{R}^3$, $t \to (2t, 1 + t^3, sin\, t + cos\, t)$, determine the equation of the tangent line to this curve at the point $t = 0$.

II M 4) Given the function $f(x, y) = (x + y)e^{x+y}$ and the unit vector $v = (cos\, \alpha, sin\, \alpha)$, determine the values of α for which the directional derivative $\mathcal{D}_v f(0, 0)$ is equal to zero and then, for any found α, calculate $\mathcal{D}^{(2)}_{v,v} f(0, 0)$.