Chapter 2

Harrod: the father of all growth models (but one)

Most of modern growth theory can be seen as a long and largely unsuccessful attempt to overcome the problems left behind by the growth model proposed by Roy Harrod () in 1939 and 1948. A similar model was proposed by Evsey Domar (1914-1997) in 1947. Harrod is a more complicated (and sometimes obscure) author but his version of the model more puzzling. Domar is more linear and in more open continuity with Keynes too. We shall not attempt here any philological exegesis of the models and content ourselves to follow the way it have been received by the literature[1].
Given its simplicity let us start with Domar's version which we shall later compare with Harrod's own. In our final comments we shall sometimes indulge in using the synthetic term Harrod-Domar model.
We shall consider a closed economy without government.

2.1. Domar the dual

In Keynes gross investment (I) is the main determinant of aggregate demand (AD) along autonomous consumption (C_a), government spending (G) and exports (E).[2] In a simple closed, private economy the level of aggregate demand and gross output (Y) depend on gross investment, given the marginal propensity to consume (c):

$$Y = \frac{1}{c} I \qquad (1)$$

In a more sophisticated open economy with government we get:

$$Y = \frac{1}{1 - c(1-t) + m} (C_a + I + G + E) \qquad (2)$$

where t is the average tax rate, m is the marginal propensity to import.
Given productive capacity, the level of aggregate demand determines the degree of capacity utilization (u) of capacity. In the short period, defined as the period in which the capital stock (productive capacity) is given, investment is therefore a main determinant of the degree of capacity utilization and, therefore, of employment. Keynes regarded investment as an unstable component of aggregate demand ad looked at government spending as a stabilization component.

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[1] On the original inspirations of Harrod's model see e.g. Kregel (1980). Others?
[2] We mainly refer to private gross investment which directly creates (or maintains) output capacity; public gross investment creates (or preserve) infrastructural capacity which only indirectly enhances (or preserve) output capacity. We therefore include public investment in government spending along government consumption.

By definition (net) investment is an increment of the capital stock ($I = \Delta K$), in other words, the determinant of productive capacity. Domar's model may be seen as an extension of Keynes' theory to the long run when the capital stock is let to vary as a result of *net* investment. We can then see here a sort of paradox: while, on the one hand, higher investment qua a determinant of aggregate demand will lead to a higher degree of current capacity utilization, on the other hand qua additional capital stock it will by definition lead to a larger productive capacity. So while investment may alleviate present troubles on the demand side, it will possibly increase future problems on the supply side. As Domar (1947, pp. 49-50) put it:

> The economy finds itself in a serious dilemma: if sufficient investment is not forthcoming today, unemployment will be here today. But if enough is invested today, still more will be needed tomorrow. … So that far as unemployment is concerned, investment is at the same time a cure for the disease and the cause of even greater ills in the future.

The question Domar faced was therefore the following: is it there an equilibrium rate of growth of the economy at which, considered the double role of investment, aggregate demand and aggregate supply grow in equilibrium (that is $\Delta Y_D = \Delta Y_S$)?

After Harrod let us call this equilibrium growth path "warranted rate of growth".

On the supply side, a *net* investment level *I* creates new capacity *ΔY* equal to

$$\Delta Y_s = \sigma I \qquad (3)$$

where σ is a technical coefficient that represents the units of potential output (or capacity) per unit of investment. For instance if σ is 0.33, ten units of investment determine three units of potential output.[3]

The incremental demand side effect of investment is captured by the standard Keynes' multiplier

$$\Delta Y_D = \frac{1}{s}\Delta I \qquad (4)$$

The economy will proceed in equilibrium if $\Delta Y_D = \Delta Y_S$. Note that while the *absolute* level of net investment (*I*) determines ΔY_S, the economy needs an *increase* in net investment over the preceding period to bring about a ΔY_D that matches ΔY_S. The equilibrium between the growth of aggregate demand and supply $\Delta Y_D = \Delta Y_S$ is where:

$$\frac{1}{s}\Delta I = \sigma I \qquad (5)$$

or

$$\frac{\Delta I}{I} = s\sigma \qquad (6)$$

[3] That fixed capital investment is larger than expected output in a given period (say one year) is not surprising since fixed capital last many years so that, during its entire life, successful investment give place to a multiple (more than proportional) amount of output.

The meaning of Domar's warranted growth rate equation is simple and inspiring: a level of net investment I determines additional capacity ΔY. In order that new capacity is employed, aggregated demand must increase by $\Delta Y_D = \Delta Y_S$, and this implies that net investment grows at a rate $\Delta I/I = s\sigma$. As Domar (1947, pp. 46-47, his italics) comments:

> investment appears on both sides of the equation [(5)]; that is, it has a dual effect: on the left side it generates income via the multiplier effect; and on the right side it increases productive capacity-the σ effect. …even though investment is present on both its sides, it does not take the same form: for on the σ side we have the amount of investment as such; but on the multiplier side we have not the *amount* of investment but its annual increment or its absolute *rate of increase*. The amount of investment (always in the net sense) may remain constant [e.g. $I_{t-1} = I_t = I_{t+1=\dots]}$, or it may go up or down, but so long as it remains positive (and except for the rare case when $\sigma \leq 0$) productive capacity increases. But if income [aggregate demand] is to rise as well, it is not enough that just any amount be invested: *an increase in income is not a function of the amount invested; it is the function of the increment of investment*. Thus the whole body of investment, so to speak, increases productive capacity, but only its very top – the increment – increases national income.

Equation (6) expresses the rate of growth of investment that, taking into account the dual effect of net investment, assure equilibrium over time.

What Domar points out is, therefore, one tragedy of capitalism, its imperative of growing or dying.

2.2. Harrod the instable[4]

Harrod is in a certain sense complementary to Domar by pointing out that the warranted rate is unknown to entrepreneurs and that, even worse, the invisible hand is unable to lead the economy to an equilibrium track. In fact, even if the economy were by fluke along this track, a little initial divergence would lead the economy far away from it.

Harrod approaches the question from the point of view of *gross* investment decisions of entrepreneurs (an investment function was instead absent in Domar). He does so by adopting the investment accelerator theory according to which the level of (gross) investment depends on the expected rate of growth of aggregate demand. The accelerator theory of investment is an old theory – attributed among others to the French economist Albert Aftalion (1874-1956) – which makes the level of investment decided by entrepreneurs depend on their expectations of demand. Regardless of Harrod's model, we consider the accelerator to be the best investment theory and an important integration to the extension to the long period of Keynes's theory of effective demand.

[4] The technical exposition of Harrod will follow Sen (1970) and Jones (1976).

In equilibrium in each period gross investment must match gross saving capacity – the saving supply forthcoming from a normally utilized productive capacity.[5] If this happens, the aggregate supply and demand are growing in equilibrium. Gross saving depends on the marginal propensity to save out of gross capacity income.

Harrod's model may be exposed through 3 equations (e.g. Sen 1970, Jones, 1976):

$$S = s\,Y \qquad (7)$$

$$I = v_n(Y^e - Y_{-1}) \qquad (8)$$

$$S = I \qquad (9)$$

Equation (7) expresses the fact that capacity savings originate from capacity income.

Equation (8) is the investment function and shows (gross) investment as dependent on the expected variation of aggregate demand during the current period with respect to the previous. It is a simple formulation of the accelerator model. The first term on the right-hand side is the so-called "capital coefficient" $v_n = K/Y$ and represents the amount of capital per unit of output desired by the entrepreneurs. Its role is to "translate" the expected growth of aggregate demand in investment. The subscript "n" stays for "normal" where Harrod uses the term "required". An example will clarify why we shall adopt the term normal. Suppose that $v_n = 3$. It means that if aggregate demand is expected to grow by €10bn in each ensuing year, given an existing level of capacity normally utilized, capitalists will wish to install €30bn of capacity (the increase in demand must last at least for the economic life of the installed equipment).

For the moment take equation (8) as an equilibrium condition in the good market (all output is sold if investment "absorb" all capacity saving). [6]

To solve the system, substitute equations (7) and (8) in equation (9) to obtain $sY = v_n(Y^e - Y_{-1})$, or:

$$\frac{s}{v_n} = \frac{(Y^e - Y_{-1})}{Y} \qquad (10)$$

[5] More precisely, we define *capacity saving* as the level of saving forthcoming from normal income, in turn the level of income obtained from a normally utilised productive capacity when all output is sold at normal prices. We shall refine these definitions during the exposition. What normal degree of capacity utilization means will be defined shortly; for the while you may think of the normal degree as full degree of utilization.

[6] This is Harrod's summing up of his equations: "The axiomatic basis of the theory which I propose to develop consists of three propositions, namely: (a) that the level of a community income is the most important determinant of its supply of saving; (b) that the rate of increase of its income is an important determinant of its demand for saving; and (c) that demand is equal to supply. It thus consists in a marriage of the 'acceleration principle' and the 'multiplier' theory" (1939, p.43).

Capitalist have invested *I* expecting aggregate demand equal to Y^e. They will therefore be satisfied if actual aggregated demand Y_a is equal to that expected, that is if $Y_a = Y^e$. In which case will they be correct?

Multiply both side of equation 4 by Y/Y_e to get:

$$\frac{s}{v_n}\frac{Y_a}{Y^e} = \frac{Y^e - Y_0}{Y^e} \qquad (11)$$

where Y_0 is output at time zero. The term on the right-hand side of equation (11) is therefore the expected growth rate g^e. Suppose that $Y = Y^e$, that is, suppose that the expectations are fulfilled. Then $g^e = s/v_n$.

Look at this result in this way. Suppose that capitalists expect a rate of growth equal to s/v_n, then their expectations will be satisfied, that is $Y_a = Y^e$. In other words, if capitalists expect the warranted rate of growth

$$g_w = \frac{s}{v_n} \qquad (12)$$

and invest consistently, then the economy will grow in equilibrium at this rate. However, there is no reason why the entrepreneurs should know g_w, which is a ratio between two magnitudes, s and v_n, that are in general unknown to capitalists. So, in general, we should not expect that the economy will grow along a warranted growth rate.

In order to get a grip on the terminology we will use later, let's take a moment to look at the *capital coefficient* $v = K/Y$. This is the capital/product ratio desired by entrepreneurs when making an investment. For example if $v = 4$, it means that they normally want to hold 4 units of capital (stock) for each unit of product (flow, e.g. annual product). Remember that capital lasts many production cycles, so although the capital stock may be worth more than the annual output, it will be depreciated (i.e. recovered) over many cycles. The desired coefficient *v* can also be called *normal*: $v = v_n$. The normal or desired capita coefficient is the ratio between the given capital stock ($\overline{K}$) and the expected (normal) output obtained from the given capacity: $v_n = \overline{K}/Y_n$.

When the entrepreneur produces with the desired capital coefficient ($v = v_n$), also the degree of capacity utilization *u* is at the normal level. The normal degree of capacity utilization is defined as $u_n = Y_n/Y_f$, where Y_n is the expected output relative to an installed production capacity $\overline{K}$, and Y_f is the *maximum* ouput obtainable from K. Let's put ourselves in the entrepreneur's shoes. If she expects a normal output per unit of time, e.g. per year, Y_n (suppose 100), he will install a capital stock $v_n \, x \, Y_n = K$ (e.g. 4 x 100 = 400). If the maximum output obtainable from *K* is Y_f (for example 120), it means that the entrepreneur requires a normal degree of capacity utilization, in the example,

u_n = 100/120, or about 83%. The enterprise will have an average unused production capacity of 17%. For what purpose? For example to cope with sudden peaks in demand and not leave an unexpected increase in customers' demand unsatisfied.

Note that if given the capital stock ($\overline{K}$) actual output is lower than expected normal output $Y_a < Y_n$, the actual degree of capacity utilization will be below normal ($u_a < u_n$). This implies that $v_a > v_n$, where $v_a = \overline{K}/Y_a$ is the actual capital coefficient. This is intuitive: if output is lower than expected and the degree of capacity utilization is below normal, the installed capital stock will be "excessive" relatively to actual output.

Since these authors do not distinguish between full operation and normal operation of capacity, in the discussion of the Harrod-Domar and Cambridge models, we will assume $Y_n = Y_f$, so $u_n = 1$ (normal and full capacity utilization coincide). We will remove this hypothesis later because the existence of a "spare capacity" can play an important role in making the system flexible.

2.3. The Harrodian instability

Presumably, entrepreneurs do not know the warranted growth rate. We might nevertheless wander whether the markets' invisible hand will lead the economy to gravitate towards this equilibrium rate. From the definitions of g_e e di g_a, respectively:

$$g_e = \frac{Y^e - Y_0}{Y^e} = 1 - \frac{Y_0}{Y^e}$$

and

$$g_a = \frac{Y_a - Y_0}{Y_a} = 1 - \frac{Y_0}{Y_a},$$

We obtain:

$$Y^e = \frac{Y_0}{(1 - g_e)}$$

and

$$Y_a = \frac{Y_0}{(1 - g_a)}$$

or:

$$\frac{Y_a}{Y^e} = \frac{(1 - g_e)}{(1 - g_a)} \quad (15)$$

Recalling from equation (11) that

$$\frac{s}{v_n}\frac{Y_a}{Y^e} = \frac{Y^e - Y_0}{Y^e}$$

or

$$g_w \frac{Y_a}{Y^e} = g_e$$

which easily becomes:

$$\frac{Y_1}{Y_1^e} = \frac{g^e}{g_w} \qquad (16)$$

Using equations (15) and (16) we finally find:

$$\frac{1 - g^e}{1 - g_a} = \frac{g^e}{g_w} \qquad (17)$$

Observe that if $g_e = g_w$, then $g_a = g_e$: this simply reaffirms,that if capitalists predicts the warranted rate correcty and invest accordingly, then their expectations are fulfilled. Let us then consider the cases in which, as we expect, firms get their predictions wrong:

a) if capitalists are over-optimistic, that is if: $g_e > g_w$, we obtain $(1 - g_e) > (1 - g_a)$, and therefore $g_a > g_e$

In other words, over-optimism leads the actual growth rate to be even larger than expected. Entrepreneurs will therefore feel that they had been too pessimist and will expect an even higher rate $g'_e > g_w$. ; this even more optimistic behavior will lead to an even larger actual rate $g_a' > g_e'$, and so on and so forth. The economy will diverge upward more and more from the warranted rate.

The discrepancy between g_a and g_w can also be observed as one between the normal capital coefficient v_n and the actual capital coefficient v_a. The latter is the actual ratio K/Y that capitalists experience at the end of the year. Note that $g_a > g_w$, or $\frac{s}{v_a} = g_a > \frac{s}{v_n} = g_w$, means that $v_a < v_n$. In other words, if the economy is growing faster than the equilibrium growth rate, capitalist will find a desired capital coefficient lower than normal and conclude that they have invested too little. Their frustration can also be seen by the fact that the degree of capacity utilization will be higher than normal $u_a > u_n$. Aggregate demand and output have been higher than expected, and entrepreneurs find they have too little capacity installed. Therefore, next period they will install even more given further stimulus to aggregate demand.

b) If, on the other hand, capitalist are too pessimist, so that $g_e < g_w$, then the actual rate will be even lower than expected $g_a < g_e$, $v_a > v_n$ and $u_a > u_n$ and the economy will diverge more and more from the warranted rate in a bottomless recession. g_w

In actual, capitalists are neither to optimist or too pessimist: the question is that they just do not know the warranted rate so we should not expect the economy to grow in equilibrium.

Interestingly, after the great (marginalist) British economist John Hicks, Serrano et al. (2019) distinguish between "static" and "dynamic" stability. The first refers to whether the model gives the right signals to agents in the direction of equilibrium if the economy is, as is generally plausible, out of balance. Static stability is therefore a necessary condition for the stability of the model. Dynamic stability refers to the intensity of the adjustment which, although it is in the right direction, could for example be too intense with an overshooting effect, or oscillation around the balance (the classic example is the spider's web theorem). In this respect, Harrod's model is statically unstable, contrary to a widespread thesis that instability would result from too intense a reaction by entrepreneurs when production capacity is not used at a normal level.

2.4. Discussion

Many years ago Franklin Serrano (1995b: 69) pointed out that although "Harrod's analysis was meant to provide 'the marriage of the acceleration principle and the multiplier theory' (Harrod 1939, p.) what he… actually accomplished was to answer to the question 'under which conditions would we obtain Say's Law?'" (see also Serrano et al. 2019). In other words the Harrod-Domar model can be intended as simply expressing the dynamic equilibrium of the economy that, if known to the entrepreneurs, will lead them to take investment decisions adequate to capacity saving, validating Say's Law that all capacity saving is invested. The surprise was that this equilibrium was unique, unknown to investors, and that the invisible hand does not lead to it.

There are two cases in which, however, we might expect the economy to follow g_w. One is if capitalists behave following Say's Law: if capitalists systematically invest all their savings, [7] the economy will actually grow at the warranted rate. Mixing Harrod and Domar, we may express this through the following two equations:

$$I = sY \qquad (18)$$

$$\Delta Y = \sigma I \qquad (19)$$

Equation (18) is the investment function that imposes that investment decisions are equal to capacity saving. Equation (19) shows the growth of capacity output as a function of investment. Substituting equation (18) in (19) it is easily obtained:

$$g_I = s\sigma$$

[7] This is the way in which Ricardo, for instance, believed in Say's Law. In actual, Ricardo believed that capitalists save in order to invest, so that investment is by definition equal to capacity savings (he of course assumed that workers do not save being too poor).

Possibly capitalists meet and agree to follow Say's Law. But, as Kalecki put it, capitalists do many things as a class, but they do not invest as a class.

The other case is that capitalists defer to a planning office the calculation of g_w, which is then imposed to the capitalist class as the mandatory rate of capital accumulation. Harrod's model was indeed quite popular in the 1950s when some mixed-economies like India or France tried some economic planning. In general, however, capitalist will refuse investment decisions imposed from outside.

But this is not all. According to Harrod's model, a market economy not only will not, in general, grow at an equilibrium rate, but it will also progressively diverge from the warranted rate. Indeed, we should expect that whenever the economy is out of equilibrium – that is, output is larger or smaller than aggregate demand – capitalists try to adjust their decisions. Precisely this behavior will led the economy to diverge from the equilibrium even more. As we shall shortly see, when, for instance, aggregate demand is larger than current output, firms will revise their expectation upward, giving a further stimulus to aggregate demand and determining an even larger disequilibrium.

To neoclassical economists the Harrod-Domar model left other problems. Even if entrepreneurs correctly guess the warranted rate, there is no reason why this should be a full-employment path. This is not a problem, of course, for heterodox scholars. Neoclassical economists want however to show that laissez-faire economies grow at full-employment. A less malevolent interpretation is the following. In general, we do not see middle or high-income economies with persisting excesses of unemployed labour force (at least not excess of unsubsidized labour population).

Neoclassical economists interpret this stylized fact as the result of the adaptation of the growth rate of output to that of population, so that excess labour is re-absorbed via wage flexibility. Conversely, heterodox economists share the classical economists' point of view that it is labour population growth that tends to adjust to economic growth and labour demand, not the other way round. In the short-run this happens through migrations and by resorting to the buffer stock consisting of marginal labour figures such as women and very young or older workers. In the long-run demographic changes may occur in fertility and mortality rates. Be it as it may, neoclassical economists are disappointed that the warranted rate is different from a given exogenous growth rate of the labour force.[8]

2.5. Conclusions

[8] The reader may also note that a rise of the saving propensity increases the warranted rate. This is a feature of the Harrod-Domar model that might please marginalist economists and be regretted by the heterodox.

The formal similarity of the warranted rate equations (6) and (12) is easily obtained by observing that while σ is the increase in output per unit of investment, v_n is the required number of investment per unit of extra output, so that $\sigma = 1/v_n$.[9] In more substantial terms, merit of Domar's model is to show the voracious insatiability of the market economy that needs a growing amount of investment to keep the installed plant busy. As this were not enough, Harrod's model showed the instability of this model of capitalism.

The last result led less-conventional economists to suggest that indicative economic planning might be helpful. Alternatively, economic policies, fiscal and monetary may avoid that the economy overheats or over-freezes. For instance, if the economy sees its growth rate progressively falling, an expansionary fiscal policy might sustain aggregate demand reverting investors' expectations from a progressive pessimism to optimism. A lax monetary policy may contribute by sustaining autonomous consumption. Although Harrod's model has been felt unsatisfactory since real economies do not show the violent kind of instability predicted by the model, on the base of the model the relative stability seems therefore due to stabilization policies. The Greek experience in the first half of the last decade suggest that once these policies are withdraw (in favour of destabilizing austerity policies), the fall of GDP can be dramatic.

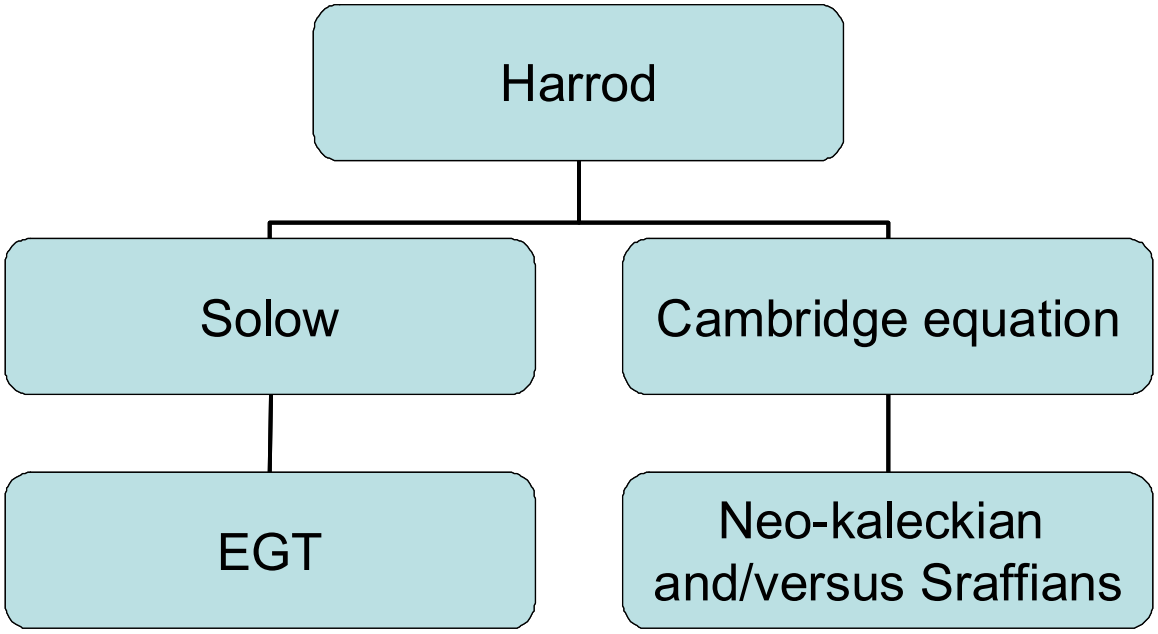
Be this as it may, in the 1950s a group of close followers of Keynes tried to prove that a market economy may find a way to stable Harrodian growth rate even without exogenous policy guidance or assistance. This attempt might appear rather strange now, since Harrod' model has a non-Keynesian feature that, as we shall see, these *post-Keynesian* authors (as they are sometimes labeled) do not amend. Simply consider the warranted growth equation $g_w = s/v_n$ and note that the equilibrium rate is positively associated to the saving propensity. This result, and the associated prescription to keep real wage low to sustain the capitalists' higher propensity to save, sound quite a non-Keynesian implications. This is why in the last couple of decades this post-Keynesian attempt has been abandoned and substituted by new non-conventional approaches considered later in this lecture notes.

Turning to neoclassical economists, they were not ready to the idea that market economies could not grow in equilibrium without the State visible hand assisting them. We shall see that while in 1956 Robert Solow pretended to solve the problem of the invisible hand driving the economy to a stable (and full employment) growth path, he also paid the tribute of losing the positive relation between the saving rate and the warranted growth rate in equation (6) or (12).

In the next chapters we shall focus upon both the Solowian and the Post-Keynesian approaches and on their more recent developments. One basic difference between the two approaches is the following.

[9] Both coefficients assume that the newly installed investment is normally utilized.

Suppose that $g_w \neq n$, where n is the rate of growth of the labour force. Solow shows that the adjustment is such that $g_w \rightarrow n$ in virtue of an adjustment in the denominator of equation (12), that is in v (while s is given) Post-Keynesian economists believe instead that the rate of capital accumulation g_k is decided by capitalists and for no reason this rate is equal to n. The early generations of post-Keynesian economists argued that, once capitalists have decided g_K, it is the numerator of equation (12), that is the saving propensity s, that will change in order that $g_w -> g_a$ (while v_n is given). This hypothesis will be later criticised by most recent heterodox schools.



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